Oka symposium, Dec 2, 2018

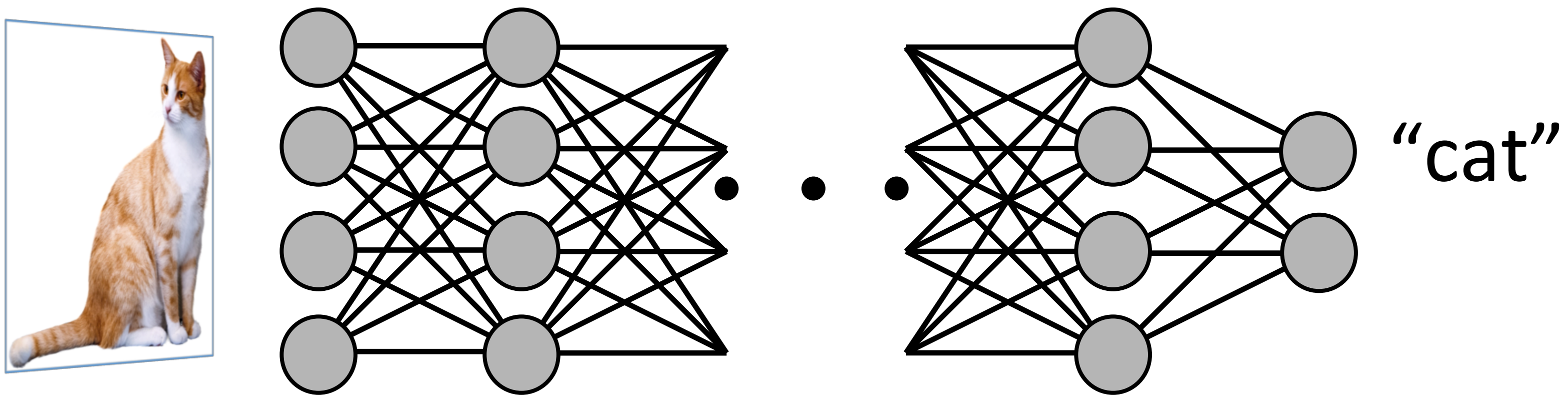
Deep Learning and AdS/CFT

Koji Hashimoto (Osaka u)

ArXiv:1802.08313, 1809.10536

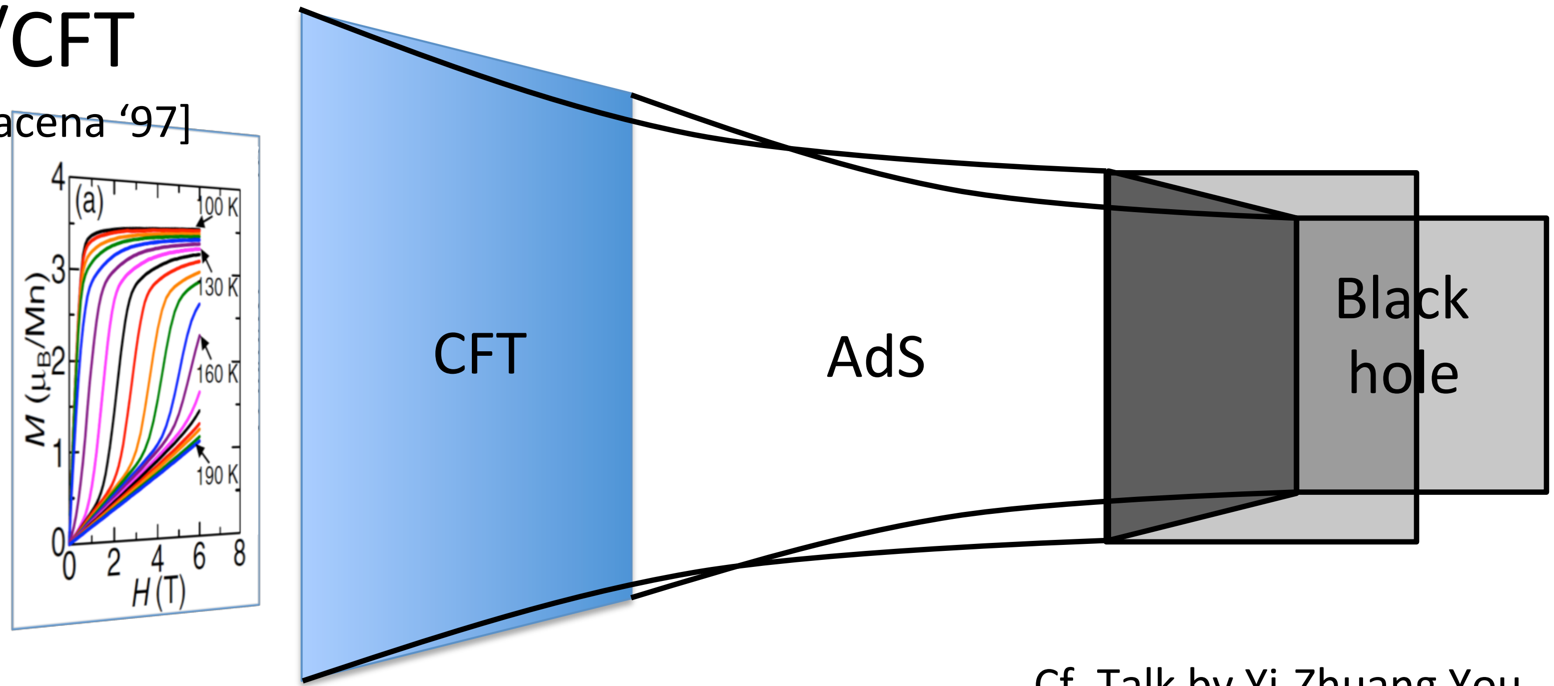
w/ S. Sugishita (Osaka),
A. Tanaka (RIKEN AIP),
A. Tomiya (RIKEN BNL)

Deep Learning



AdS/CFT

[Maldacena '97]



Cf. Talk by Yi-Zhuang You

1. Formulation of AdS/DL correspondence

2. Deeply learning QCD

1. Formulation of AdS/DL correspondence

1-1 **Three motivations**

review **AdS/CFT: quantum response from geometry**

review **Deep learning: optimized sequential map**

1-2 **From AdS to DL**

1-3 **Dictionary of AdS/DL correspondence**

1-1 (1) Brief history of quantum gravity

1974 Yoneya, Scherk-Schwarz: String = quantum gravity.

Yoneya, Prog.Theor.Phys. 51 (1974) 1907.
Scherk, Schwarz, Nucl.Phys. B81 (1974) 118.

1976 Hawking: Information loss problem of black holes.

Hawking, Phys.Rev.D14(1976)2460.

1997 Maldacena: Discovery of AdS/CFT.
A quantum gravity is nonperturbatively defined.

Maldacena, Adv.Theor.Math.Phys. 2 (1998) 231.

2002 Holographic QCD.

Karch, Katz, JHEP 0206:043.
Kruczenski,Mateos,Myers,Winters JHEP 0405:041.
Sakai, Sugimoto, PTP 113 (2004) 843.

2008 Holographic superconductor.

Hartnoll, Herzog, Horowitz, PRL 101(2008)031601.

2009 Bulk reconstruction.

Heemskerk,Penedones,Polchinski,Sully, JHEP 0910:079.

1-1 Emergent geometry?

Emergence of AdS radial direction?

Bulk reconstruction and locality.

[Heemskerk, Penedones, Polchinski, Sully 09]

Entanglement entropy reconstruction.

[Balasubramanian, Chowdhury, Czech, de Boer, Heller 13]
[Myers, Rao, Sugishita 14]

Optimization of boundary path integral.

[Caputa,Kundu,Miyaji,Takayanagi,Watanabe 17]

Renormalization and effective LG theory.

[Ki-Seok Kim, Chanyong Park 16]

AdS/MERA. [Swingle 12]

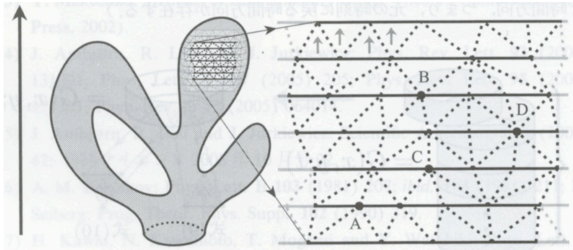
Emergence of smooth neural network space?

Statistical neural network. [Amari et al.]

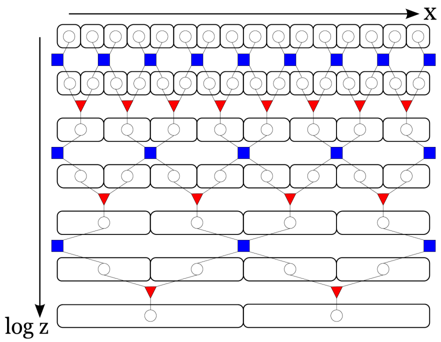
1-1 Discretized QG spacetime?

Quantum gravity, discretized

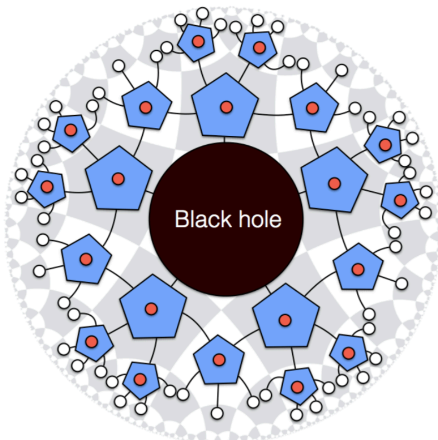
Causal dynamical triangulation [Ambjorn, Loll 1998]



AdS/MERA
[Swingle 2009]



HaPPY code
[Pastawski, Yoshida, Harlow, Preskill 2015]



1-1 (2) Solving inverse problem

AdS/CFT
(No proof, no derivation)

Classical gravity
in d+1 dim. spacetime

||

Quantum field theory
in d dim. spacetime
(Strong coupling limit,
large DoF limit)

Conventional holographic modeling

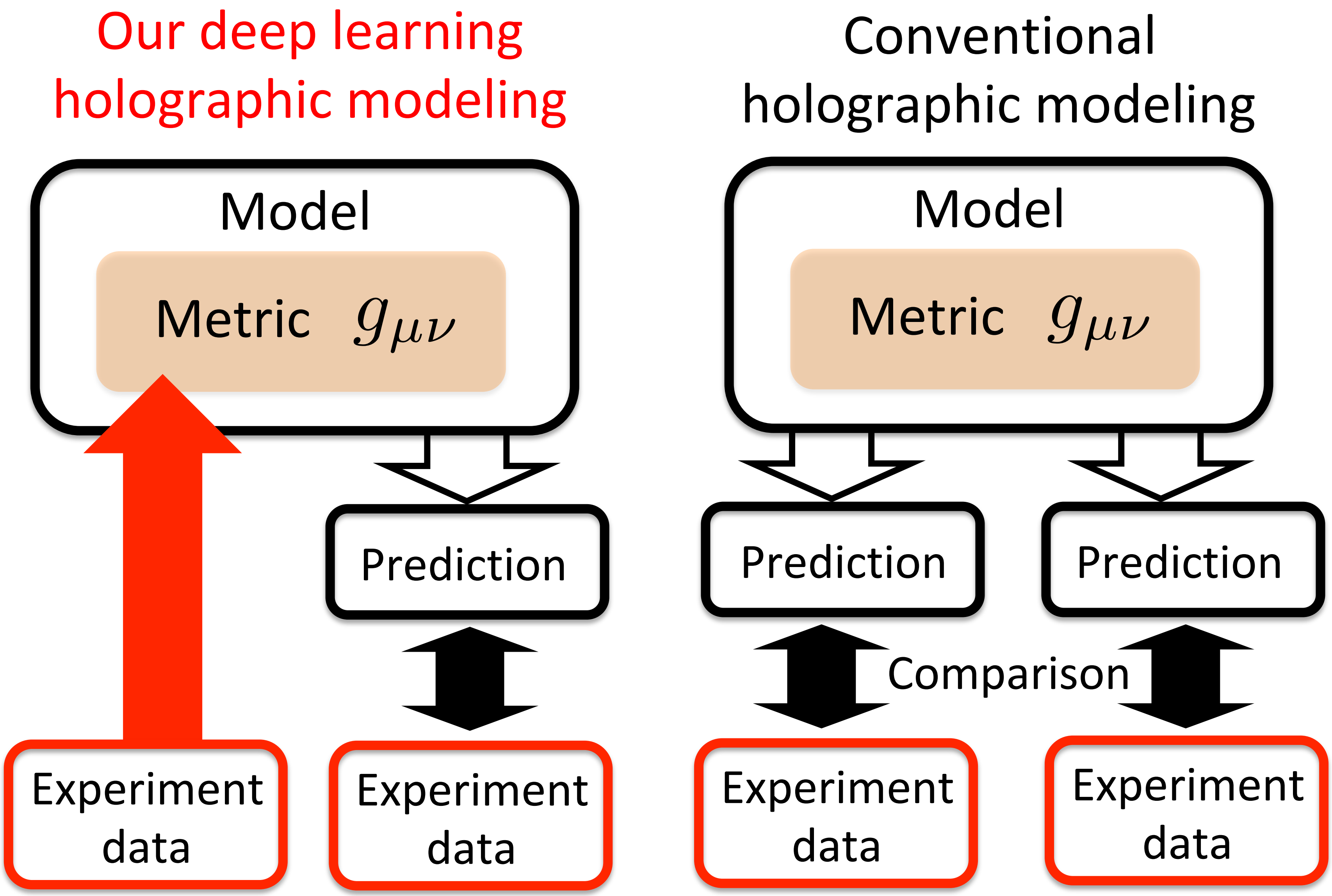
Model
Metric $g_{\mu\nu}$

Prediction | Prediction

Comparison

Experiment data | Experiment data

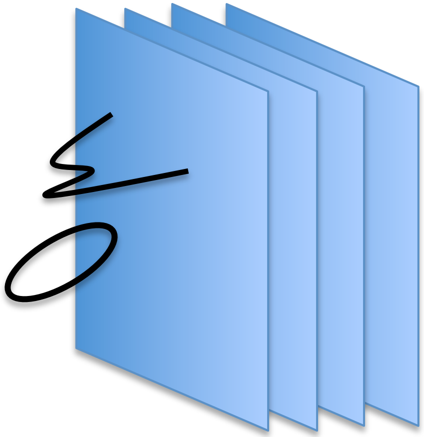
1-1 (2) Solving inverse problem



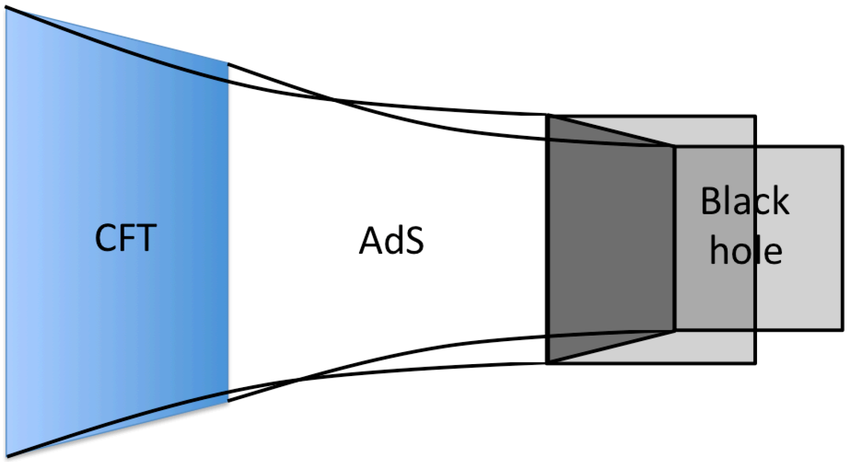
1-1

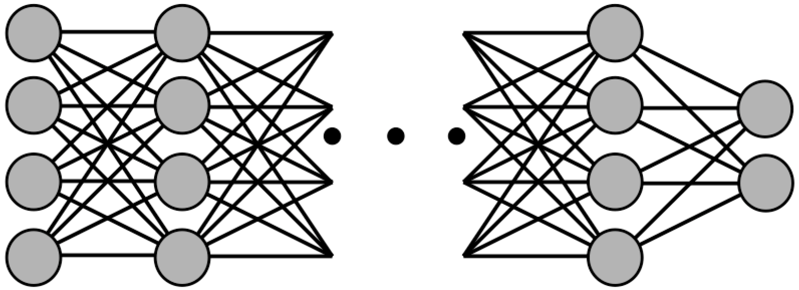
(3) Most fundamental principle?

Brane
(Superstring theory)



Brain
(Neuroscience)





review

AdS/CFT: quantum response from geometry

[Klebanov, Witten]

Classical scalar field theory in (d+1) dim. geometry

$$S = \int d^{d+1}x \sqrt{-\det g}\left[(\partial_\eta \phi)^2 - V(\phi)\right]$$

$$ds^2 = -f(\eta)dt^2 + d\eta^2 + g(\eta)(dx_1^2 + \cdots + dx_{d-1}^2)$$

- AdS boundary ($\eta \sim \infty$) : $f \sim g \sim \exp[2\eta/L]$
- Black hole horizon ($\eta \sim 0$) : $f \sim \eta^2,\ g \sim \text{const.}$

Solve EoM, get response $\langle \mathcal{O} \rangle_J$. Boundary conditions:

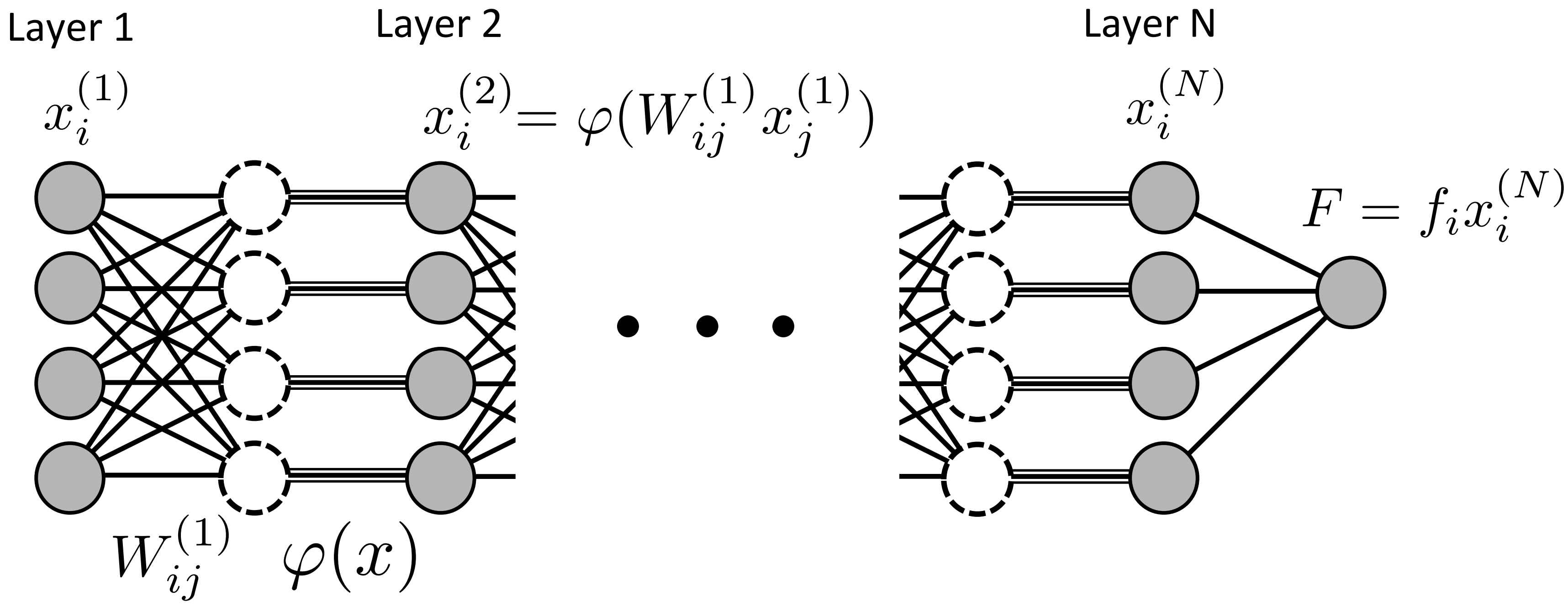
- AdS boundary ($\eta \sim \infty$) :

$$\phi = J e^{-\Delta_- \eta} + \frac{1}{\Delta_+ - \Delta_-} \langle \mathcal{O} \rangle e^{-\Delta_+ \eta}$$

- Black hole horizon ($\eta \sim 0$) : $\partial_\eta \phi \,\Big|_{\eta=0} = 0$

Deep learning : optimized sequential map

review



"Activation function" (fixed nonlinear fn.)

"Weights" (variable linear map)

1) Prepare many sets $\{x_i^{(1)}, F\}$: input + output
2) Train the network (adjust W_{ij}) by lowering
   "Loss function" $E \equiv \sum_{\mathrm{data}} \left| f_i(\varphi(W_{ij}^{(N-1)} \varphi(\cdots \varphi(W_{lm}^{(1)} x_m^{(1)})))) - F \right|$

1-2 From AdS to DL

Bulk EoM

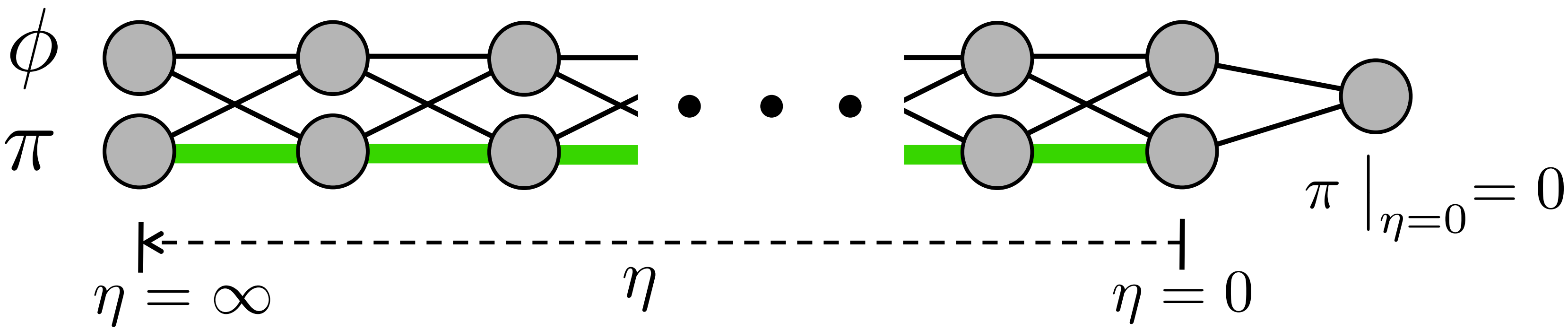
$$\partial_\eta^2 \phi + \underline{h(\eta)}\partial_\eta \phi - \frac{\delta V[\phi]}{\delta \phi} = 0$$

metric $\quad h(\eta) \equiv \partial_\eta \left[\log \sqrt{f(\eta) g(\eta)^{d-1}} \right]$

Discretization, Hamilton form

$$\begin{cases} \phi(\eta + \Delta\eta) = \phi(\eta) + \Delta\eta\, \pi(\eta) \\ \pi(\eta + \Delta\eta) = \pi(\eta) + \Delta\eta \left(h(\eta)\pi(\eta) - \dfrac{\delta V(\phi(\eta))}{\delta \phi(\eta)} \right) \end{cases}$$

Neural-Network representation



1-3 Dictionary of AdS/DL correspondence

AdS/CFT	Deep learning
Emergent space $\infty > \eta \geq 0$	Depth of layers $i = 1, 2, \cdots, N$
Bulk gravity metric $h(\eta)$	Network weights $W_{ij}^{(a)}$
Nonlinear response $\langle \mathcal{O} \rangle_J$	Input data $x_i^{(1)}$
Horizon condition $\partial_\eta \phi \,\big|_{\eta=0} = 0$	Output data F
Interaction $V(\phi)$	Activation function $\varphi(x)$

1. Formulation of AdS/DL correspondence

1-1 **Solving inverse problem**

review **Deep learning : optimized sequential map**

review **AdS/CFT: quantum response from geometry**

1-2 **From AdS to DL**

1-3 **Dictionary of AdS/DL correspondence**

1. Formulation of AdS/DL correspondence

2. Deeply learning QCD

2. Deeply learning QCD

2-1 **Demonstration of holographic modeling**

2-2 **Deeply learning QCD**

1) Use a QCD data.
2) Let the network learn the metric.
3) Calculate other physical quantities.

2-1 Demonstration of holographic modeling

Model

Metric $g_{\mu\nu}$

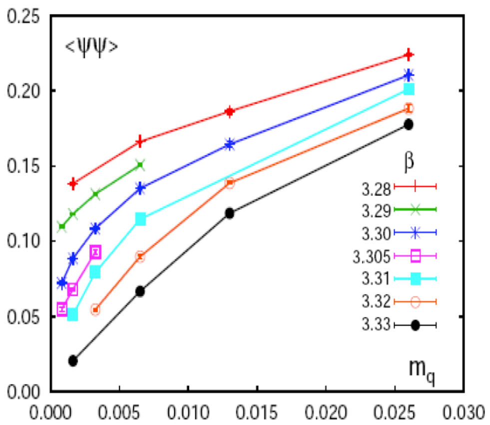
Deep Learning

Experiment data

Prediction

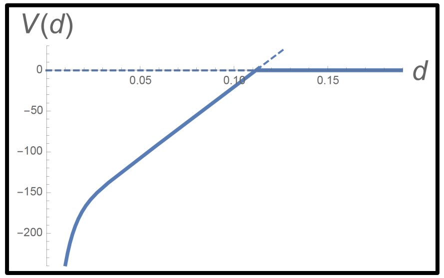
Experiment data

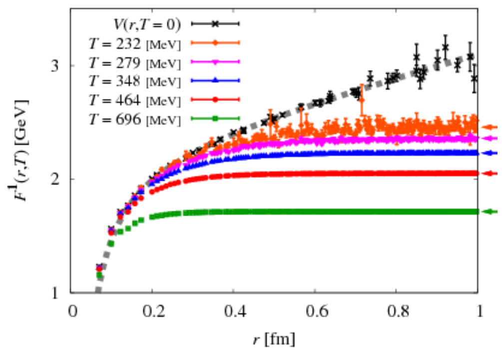
Lattice QCD data:
chiral condensate
VS quark mass



[RBC-Bielefeld collaboration, 2008]
(Courtesy of W.Unger)

Q Qbar potential





[T.Ishikawa et al., 2008,
CPPACS + JLQCD collaboration]

2-2 Deeply learning QCD

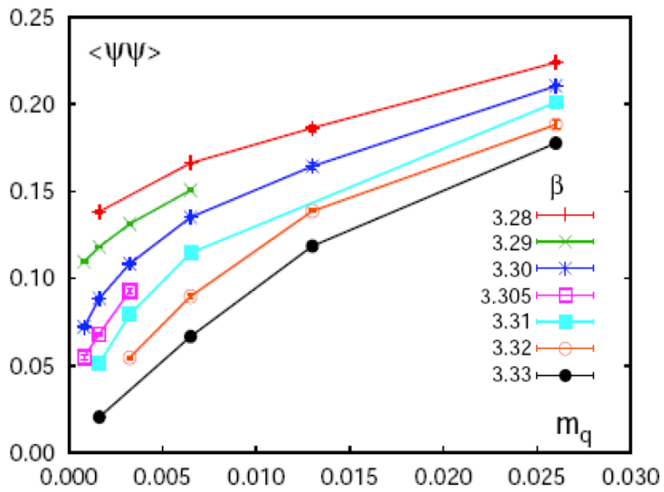
1) Use a QCD data.
2) Let the network learn the metric.
3) Calculate other physical quantities.

2-2

Deeply learning QCD

1) Use a QCD data.
2) Let the network learn the metric.
3) Calculate other physical quantities.

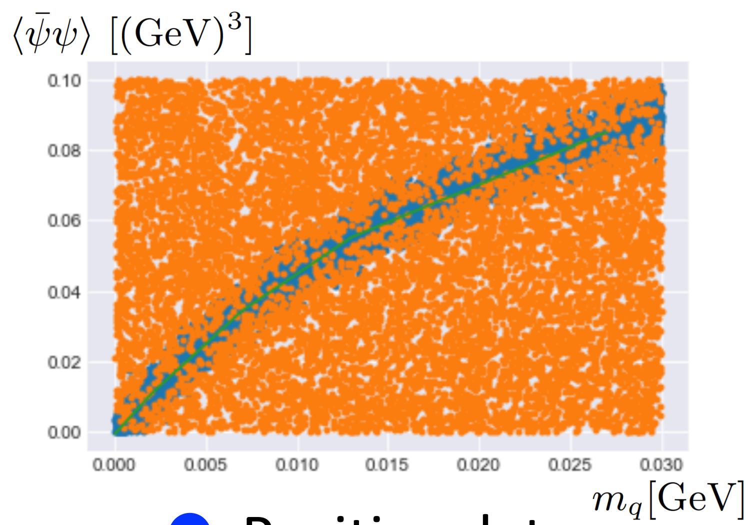
Chiral condensate VS quark mass.



β=3.30 ⇔ T=196[MeV]
[RBC-Bielefeld collaboration, 2008]
(Courtesy of W.Unger)

Pick up

β=3.33 data



Positive data
Negative data

2-2 Deeply learning QCD

1) Use a QCD data.
2) Let the network learn the metric.
3) Calculate other physical quantities.

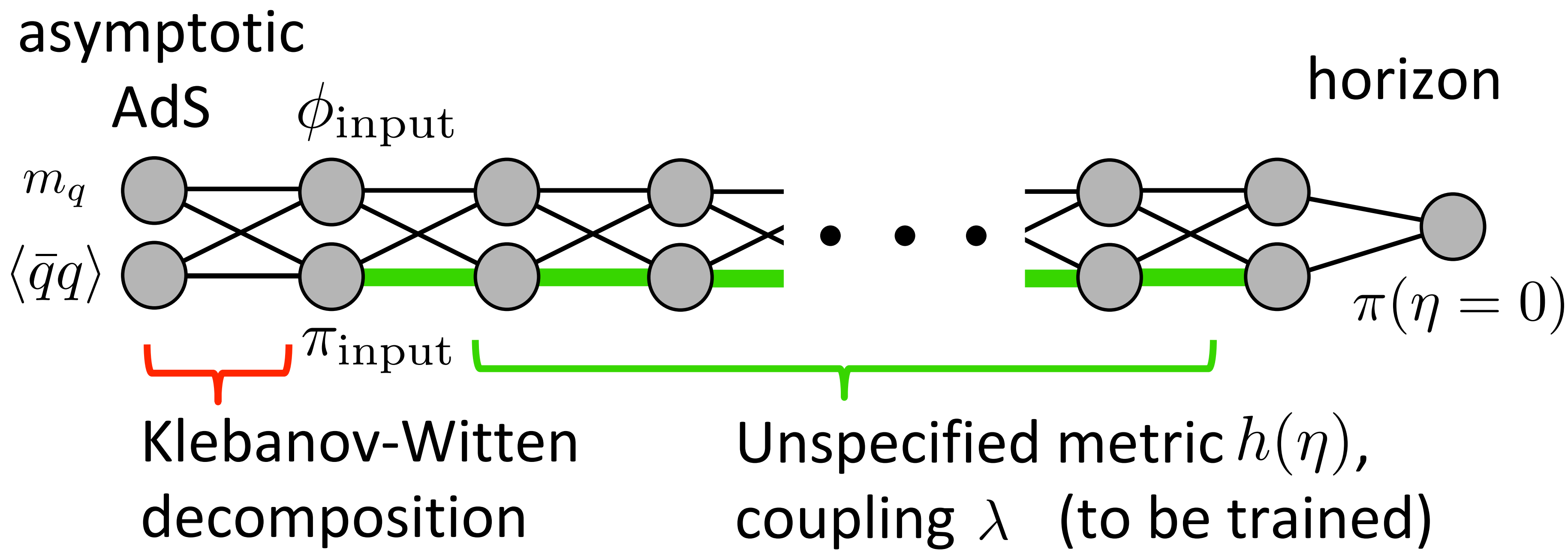
Map it to asymptotic scalar configuration. [Klebanov, Witten] [DaRold,Pomarol][Karch,Katz,Son,Stephanov] [Cherman,Cohen,Werbos]

$$\phi = \frac{\sqrt{N_c}}{4\pi} m_q e^{-\eta} + \frac{\pi}{2\sqrt{N_c}} \langle \bar{q} q \rangle e^{-3\eta} - \frac{\lambda}{2} \left(\frac{\sqrt{N_c}}{4\pi} m_q \right)^3 \eta e^{-3\eta}$$

- Conformal dimension of $\langle \bar{q} q \rangle$ is 3.
- Sub-leading contribution, present.
- Everything measured in unit of AdS radius.

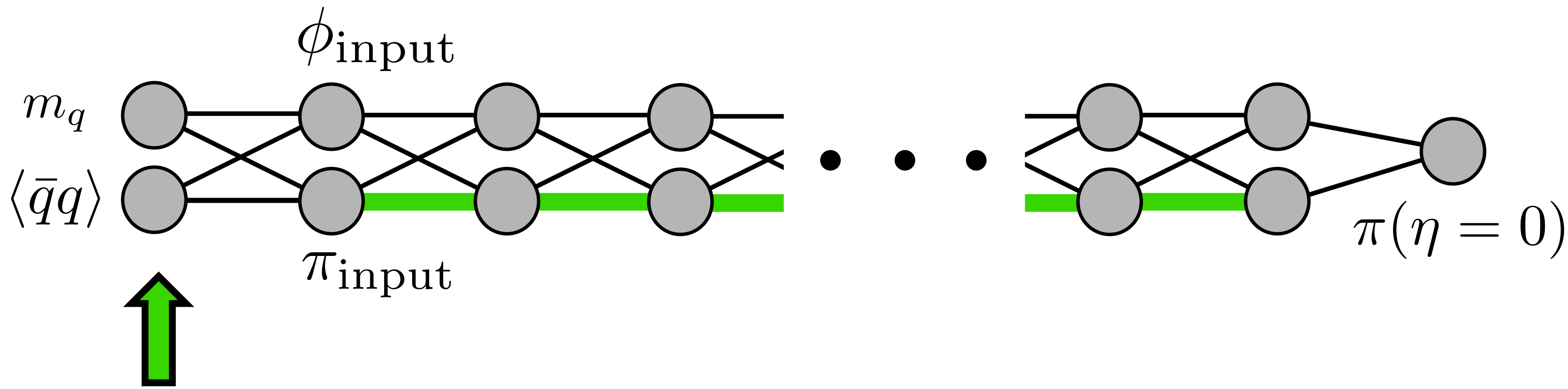
2-2 Deeply learning QCD

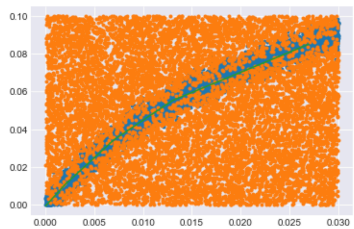
1) Use a QCD data.
2) Let the network learn the metric.
3) Calculate other physical quantities.



2-2 Deeply learning QCD

1) Use a QCD data.
2) Let the network learn the metric.
3) Calculate other physical quantities.



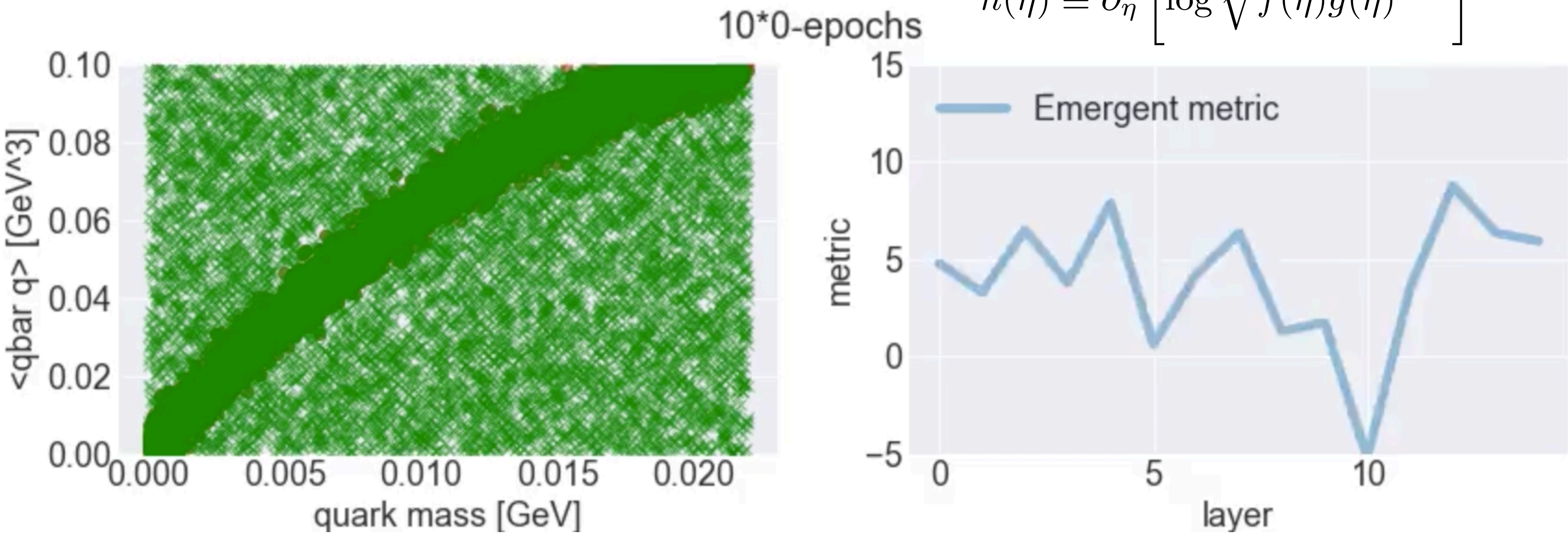


QCD lattice data

2-2 Deeply learning QCD

1) Use a QCD data.
2) Let the network learn the metric.
3) Calculate other physical quantities.

$$h(\eta) \equiv \partial_\eta \left[\log \sqrt{f(\eta) g(\eta)^{d-1}}\right]$$



Learned value of (AdS radius)$^{-1}$: $1/L$ = 237(3)[MeV]

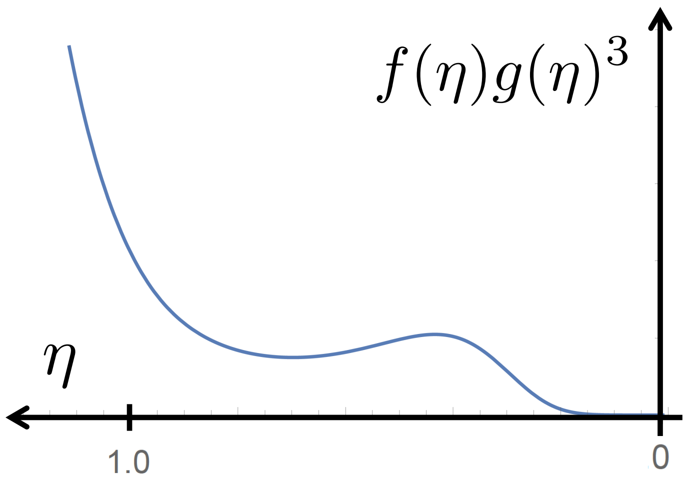
bulk coupling : λ/L = 0.0127(6)

2-2

Deeply learning QCD

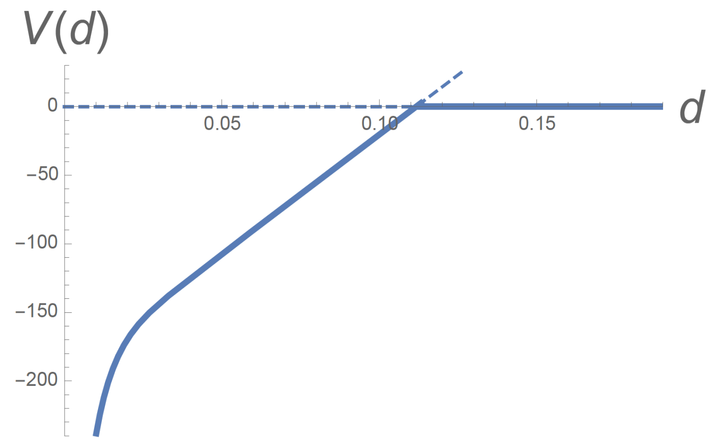
1) Use a QCD data.
2) Let the network learn the metric.
3) Calculate other physical quantities.

Learned metric



Procedures based on [Maldacena] [Rey,Theisen,Yee]

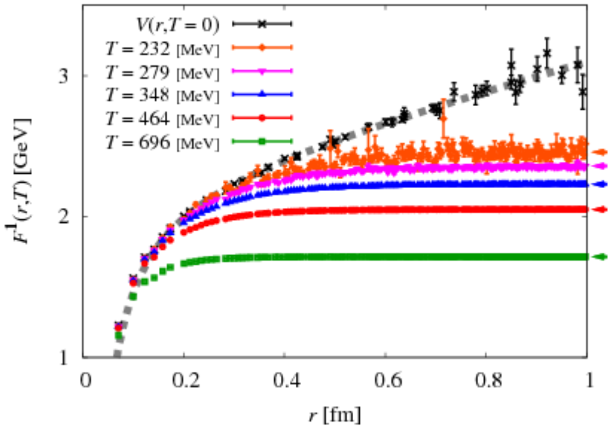
Q Qbar potential

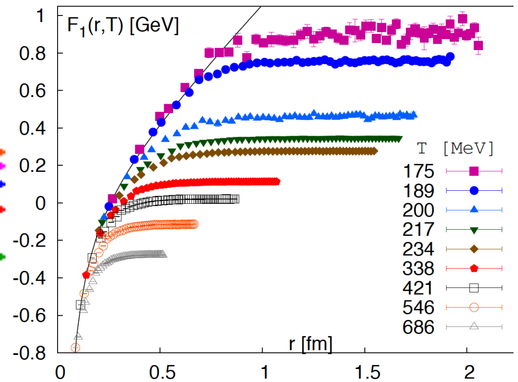


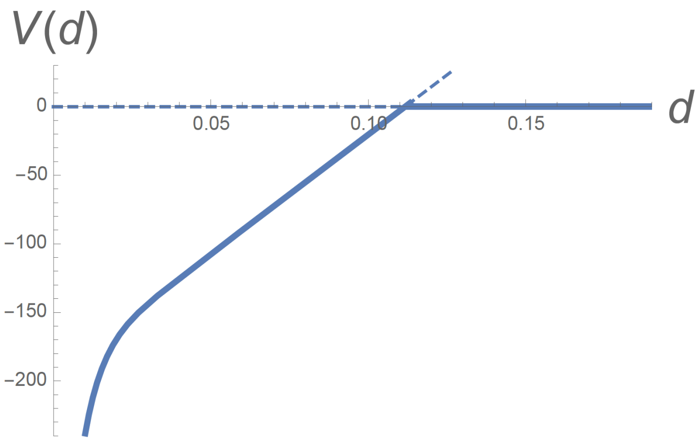
2-2 Deeply learning QCD

1) Use a QCD data.
2) Let the network learn the metric.
3) Calculate other physical quantities.

Q Qbar potential







[T.Ishikawa et al., 2008, CPPACS + JLQCD collaboration]

[Petreczky, 2010]

2-1 Demonstration of holographic modeling

Model

Metric $g_{\mu\nu}$

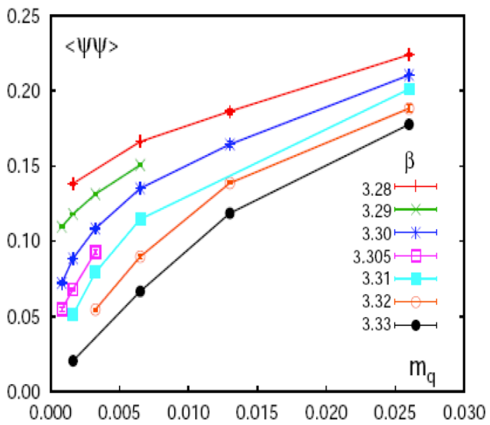
Deep Learning

Prediction

Experiment data

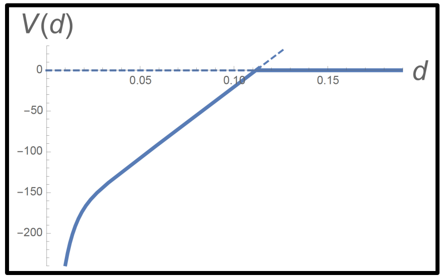
Experiment data

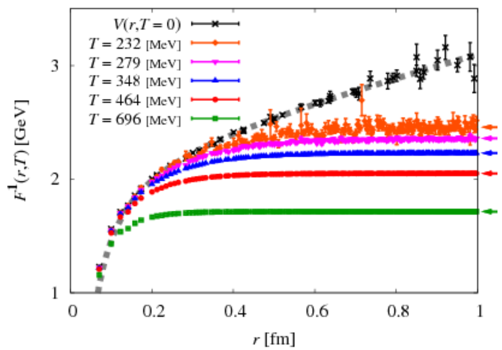
Lattice QCD data:
chiral condensate
VS quark mass



β=3.30 ⇔ T=196[MeV]
[RBC-Bielefeld collaboration, 2008]
(Courtesy of W.Unger)

Q Qbar potential





[T.Ishikawa et al., 2008,
CPPACS + JLQCD collaboration]

1. Formulation of AdS/DL correspondence

2. Deeply learning QCD