Richard Thompson's groups and their actions on non-positively curved spaces

加藤本子（愛媛大学）

Thompson's groups are discovered by Richard Thompson in 1960s, and have been studied in various areas of mathematics including geometric group theory, topology and applied mathematics. There are three types of Thompson's groups: F, T and V, which are defined as infinite groups consisting of homeomorphisms of $[0, 1]$, S^1 and the Cantor space, respectively. These groups are known to have many strange properties. For example, T and V are the first examples of finitely presented infinite simple groups. It is conjectured that Thompson's group F is not amenable, but the problem is still wide open.

A CAT(0) space is a geodesic space whose geodesic triangles are no fatter than the comparison triangles in $\mathbb{R}^2$, which is an analogue of Riemannian manifolds of non-positive sectional curvature. Fixed point free actions of Thompson's group F on CAT(0) spaces are studied in relation with the problem of amenability. In particular, F is known to have a proper action on an infinite-dimensional CAT(0) cube complex. By similar constructions, T and V also act properly on infinite-dimensional CAT(0) cube complexes. On the other hand, it is difficult to construct such actions on finite-dimensional CAT(0) spaces. For example, T and V always fix a point when they act cellularly on simplicial trees, which are one-dimensioanl CAT(0) cube complexes ([2]). Considering these results, it is natural to ask whether Thompson's groups T and V fix a point when they act on finite-dimensioanl CAT(0) spaces.

According to [3], we give a sufficient condition for group elements to have a fixed point when the group acts semi-simply on a k-dimensional CAT(0) space. This condition is written as intrinsic properties of a group, and can be confirmed without any information on group actions on CAT(0) spaces. As an application, we show that every element of the commutator subgroup of Thompson's group F always fixes a point when F acts semi-simply on a finite-dimensional complete CAT(0) space. It follows that Thompson's group T and V have a global fixed point when they act semi-simply on a finite-dimensional complete CAT(0) space.

References

[1] D. S. Farley, *Actions of picture groups on CAT(0) cubical complexes*, Geom. Dedicata **110**, 221–242, 2005.

[2] D. S. Farley, *A proof that Thompson's groups have infinitely many relative ends*, J. Group Theory **14**, 649–656, 2011.

[3] M. Kato, *On groups whose actions on finite-dimensional CAT(0) spaces have global fixed points*, J. Group Theory **22**, 1089–1099, 2019.