

# Japan-Korea School of 3-manifolds

(November 29-December 8, 2002)

Nara Women's University

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## PROGRAM

(Most talks are held in Lecture Room B1406 in Building B.)

November 29 (Friday)

14:00–15:00 Sakuma Makoto (Osaka Univ.)

Comparing two convex hull constructions for cusped hyperbolic manifolds

(Joint work with H. Akiyoshi, M. Wada and Y. Yamashita)

(This is a joint lecture with the conference entitled "Topology and Computer", and held in Conference Room of Faculty of Science in Building A.)

15:30–16:30 Matsuda Hiroshi (Tokyo Univ., JSPS Research Fellow)

Small knots and links

**Abstract:** Let  $F$  be a closed orientable surface embedded in a compact, irreducible orientable 3-manifold  $M$ . A surface  $F$  is said to be essential if  $F$  is incompressible in  $M$  and if  $F$  is not parallel to any component of  $\partial M$ . A 3-manifold  $M$  is said to be small if there is no essential surface  $F$  in  $M$ . A knot or link  $K$  in a closed 3-manifold  $M$  is said to be small if the 3-manifold  $M - \text{int}N(K)$  is small. Alan Reid asked the following question.

**Question 0':** For every positive integer  $g$ , does there exist a closed small 3-manifold  $M$  such that the Heegaard genus of  $M$  is  $g$ ?

In order to answer Question 0', it is enough to answer the following question(s).

**Question  $n$ :** Let  $n$  be a positive integer. Does there exist an  $n$ -component small link in  $S^3$ ? A solution to Question  $n$  together with theorems due to Hatcher and Moriah-Rubinstein (or Rieck-Sedgwick) answers Question 0' for  $g = n/2$  (respectively  $g = (n + 1)/2$ ) if  $n$  is even (resp. odd).

In this talk, I answer Question  $n$  for  $n \leq 5$  by constructing (infinitely many) concrete examples of  $n$ -component small links with  $n \leq 5$  in  $S^3$ . To my best knowledge, no concrete example of an  $n$ -component small link in  $S^3$  is known yet for  $n \geq 6$ . Recently, Ian Agol announced the existence of  $n$ -component small links in  $S^3$  for every  $n$ , however, he constructs no concrete example.

## November 30 (Saturday)

### MORNING SESSION

9:30–11:30 **Ichihara Kazuhiro** (Nara Women's Univ., JSPS Research Fellow)

**On Agol's paper: Small 3-manifolds of large genus**

**Abstract:** This talk gives an explanation of the following paper.

<http://front.math.ucdavis.edu/math.GT/0205091>

### AFTERNOON SESSION

13:00–15:00 **Goda Hiroshi** (Tokyo University of Agriculture and Technology)

**Heegaard splitting for sutured manifolds and Novikov inequality for knots and links**

**Abstract:** According to the work of Pajitnov-Rudolph-Weber, I will explain an application of Novikov's circled valued Morse theory to knots and links. We can regard this Morse theory as Heegaard splitting for sutured manifolds, and then the Heegaard genus can be estimated by  $k$ -th knot invariants ( $k$ -th Alexander polynomials). Several open problems will be presented.

15:30–16:30 **Teragaito Masakazu** (Hiroshima Univ.)

**Toroidal surgeries on hyperbolic knots**

**Abstract:** For a hyperbolic knot, an exceptional surgery is conjectured to yield a Seifert fibered manifold or a toroidal manifold. In this talk, I give a survey of known results about toroidal surgery, including the very recent result by Gordon-Luecke, and my results, a conjecture and problems.

16:45–17:45 **Lee Sangyop** (Korea Institute for Advanced Study)

**Essential spheres and tori after Dehn fillings**

18:00–20:00 **PARTY (Student Union)**

## December 1(Sunday)

### MORNING SESSION

9:30–10:30 **Morimoto Kanji** (Konan Univ.)

An overview of

“Annuli in generalized Heegaard splittings and degeneration of tunnel number”

by Scharlemann-Schultens

**Abstract:** In this paper, Scharlemann-Schultens showed that

- (1)  $t(K_1\#K_2\#\dots\#K_n) \geq 1/3(t(K_1) + t(K_2) + \dots + t(K_n))$  for any prime knots  $K_1, K_2, \dots, K_n$ .
- (2)  $t(K_1\#K_2\#\dots\#K_n) \geq 2/5(t(K_1) + t(K_2) + \dots + t(K_n))$  if none of  $K_1, K_2, \dots, K_n$  are 2-bridge knots.
- (3)  $t(K_1\#K_2) \geq 2/5(t(K_1) + t(K_2))$  for any prime knots  $K_1, K_2$ .

This is proved by analyzing the intersections of the decomposing annuli and generalized Heegaard splittings of the knot exteriors. The arguments are rather complicated but very useful for studying 3-manifolds from the point of Heegaard splittings. So I will try to give a plain overview of the proof.

11:00–12:00 **Saito Toshio** (Osaka Univ.)

(1,1)-knots as viewed from curve complex

**Abstract:** J. Hempel studied Heegaard splittings of closed 3-manifolds by using the curve complex. In this paper, we apply this idea to (1,1)-splittings of genus one 1-bridge knots, and introduce the concept of the distance of a (1,1)-splitting. By studying the (1,1)-splittings of distance  $\leq 2$ , we will prove that a genus one 1-bridge knot is hyperbolic if and only if it has a (1,1)-splitting with distance  $\geq 3$  except for certain knots. Further we will prove that there are (1,1)-splittings with arbitrarily high distance.

### AFTERNOON SESSION

13:30–15:30 **Ichihara Kazuhiro** (Nara Women’s Univ., JSPS Research Fellow)

On Lackenby’s paper:

Heegaard splittings, the virtually Haken conjecture and Property tau

**Abstract:** This talk gives an explanation of the following paper.

<http://front.math.ucdavis.edu/math.GT/0205327>

## December 6 (Friday)

(This day the talks are held in Nara International Seminar House)

14:00–15:00 **Ichihara Kazuhiro** (Nara Women's Univ., JSPS Research Fellow)

### Heegaard gradient of Seifert fibered 3-manifold

**Abstract:** We show that a Seifert fibered 3-manifold has zero infimal Heegaard gradient if and only if it has the infinite fundamental group. We also give a necessary and sufficient condition for a collection of finite coverings of a Seifert fibered 3-manifold to have zero infimal Heegaard gradient.

15:30–16:30 **Tsutsumi Yukihiro** (Keio Univ., JSPS Research Fellow)

### Once-punctured torus Haken number of knots

**Abstract:**

**Theorem A:** A hyperbolic knot in a non-Haken 3-manifold bounds only 5 mutually disjoint, non-parallel, genus one Seifert surfaces.

**Theorem B:** There exists a hyperbolic knot in any non-Haken 3-manifold which bounds 3 mutually disjoint, non-parallel, genus one Seifert surfaces.

**Theorem C:** For any natural number  $n$ , there exists a genus two hyperbolic knot in any non-Haken 3-manifold which bounds  $n$  mutually disjoint, non-parallel, genus two Seifert surfaces.

## December 7 (Saturday)

### MORNING SESSION

9:30–11:30 **Motegi Kimihiko** (Nihon Univ.)

### An overview of

### “Representations of 3-manifold groups” by Culler-Shalen

**Abstract:** Let  $M$  be a compact, connected, orientable, irreducible 3-manifold. An essential surface in  $M$  gives rise to a simplicial action of  $\pi_1(M)$  on a tree  $T$  which is nontrivial (i.e., no vertex is fixed by the entire group  $\pi_1(M)$ ) and without inversions. It is not true that every nontrivial action without inversions of  $\pi_1(M)$  on a tree arises in such a manner. However, to every nontrivial action without inversions of  $\pi_1(M)$  on a tree  $T$ , one can associate an essential surface. When and how can we construct a nontrivial action without inversions of  $\pi_1(M)$  on a tree  $T$ ? I will give an overview of so called “Culler-Shalen theory” which answers this question. As applications, I will also briefly discuss “Neuwirth conjecture”, “cyclic surgery theorem”, “Smith conjecture”, “diameter of boundary slopes”.

## AFTERNOON SESSION

13:00–14:00 **Shimokawa Koya** (Saitama Univ.)

**Exceptional surgery and boundary slopes**

(with Masaharu Ishikawa (Tokyo Metropolitan Univ.) and  
Thomas W. Mattman (California State Univ., Chico))

**Abstract:** In this talk we consider the connection between cyclic, finite, and Seifert slopes and boundary slopes. We show that we can find strict boundary slopes near cyclic, finite and Seifert slopes. We also prove that the diameter of the set of strict boundary slopes can be bounded from below using the Culler-Shalen norms of those slopes.

14:30–15:30 **Hong Sungbok** (Korea University)

**Rubinstein-Scharlemann graphic for lens spaces**

16:00–17:00 **Matsuda Hiroshi** (Tokyo Univ., JSPS Research Fellow)

**Closed incompressible surfaces in the complements of knots of braid index four**

**Abstract:** Closed incompressible surfaces embedded in the complements of links of braid index three are classified by Lozano and Przytycki, and independently by Finkelstein. In this talk, we obtain some features of closed incompressible surfaces embedded in the complements of knots of braid index four. As a corollary, we show that there is no closed totally geodesic surfaces embedded in the complements of hyperbolic knots of braid index four. This partially solves a conjecture raised by Menasco and Reid.

18:00–20:00 **PARTY (Student Union)**

## December 8 (Sunday)

### MORNING SESSION

9:30–10:30 **Hayashi Chuichiro** (Japan Women's Univ.)

**An algorithm for finding a satellite diagram of a (1,1)-splitting**  
(with Hiroshi Goda and Hyun-Jong Song)

**Abstract:** We give an algorithm for finding a satellite diagram of a given slope on the standard torus for a given (1,1)-knot.

11:00–12:00 **Saito Toshio** (Osaka Univ.)

**Meridionally incompressible surfaces and (1,1)-splittings**

**Abstract:** It's an interesting problem to study how a surface in a 3-manifold  $M$  intersects a fixed Heegaard splitting of  $M$ . Haken's theorem is one of the most famous results in this direction. In this talk, by studying the intersections between a closed surface in the exterior of a (1,1)-knot  $K$  which is meridionally incompressible in  $(M, K)$  and a fixed (1,1)-splitting of  $(M, K)$ , we obtain a result similar to Haken's theorem.

# MAP

