The level truncation of pure gauge solutions in string field theory

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1. Introduction

Schnabl constructed an analytic solution with a parameter in open string field theory. For $\lambda = 1$, the solution represents the nonperturbative tachyon vacuum and for $0 < \lambda < 1$, it corresponds to a trivial pure gauge configuration. The vacuum energy of the tachyon solution was calculated in different three manners: i) analytic calculation using wedge states, ii) numerical calculation using L_0 level truncation, iii) numerical calculation using $\mathcal{L}_0$ level truncation. In the latter two cases, only the kinetic terms were evaluated and the interaction terms were not included. For the pure gauge solution, the vacuum energy was calculated by analytic method only.

We evaluate the vacuum energy of the Schnabl's solution with the parameter in terms of the L_0 level truncation up to level 6, and including interaction terms up to level (6,18). The resulting energy behaves as expected for the parameter λ. The interaction terms have an insignificant effect on the vacuum energy, although the solution in the Virasoro basis does not include the term, $\lim_{N\to\infty}\psi_N$, that is necessary for providing the correct energy in the analytic calculation.

2. Schnabl's Solution

Schnabl's solution describes the non-perturbative tachyon vacuum in Witten's cubic open bosonic string field theory (CSFT).

Witten's action

$$S = -\frac{1}{g_0^2}\left[\frac{1}{2}\langle\Psi, Q_B\Psi\rangle + \frac{1}{3}\langle\Psi, \Psi * \Psi\rangle\right]$$

Equation of motion

$$Q_B\Psi + \Psi * \Psi = 0$$

Q_B is the Kato-Ogawa BRS charge.

Schnabl proves analytically Sen's first conjecture.

Schnabl's analytic solution gave D25 brane tension.

The classical solution found by Schnabl can be written as

$$\Psi(\lambda) = \lim_{N\to\infty}\left(\lambda^N\psi_N - \sum_{N=2}^{N+2}\lambda^{n-1}\partial_n\psi_{n-2}\right)$$

where if $N \longrightarrow \infty$, the first term disappears.

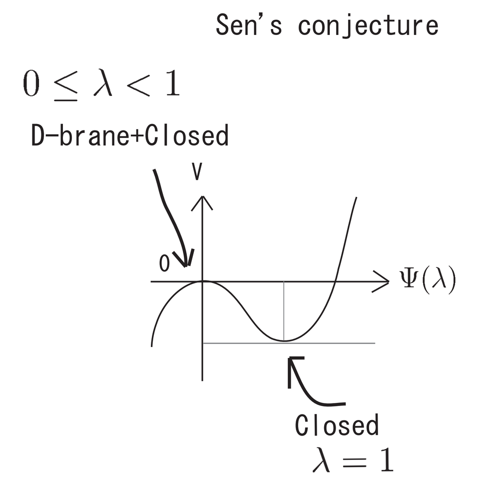
$$\psi_{n-2} = \frac{1}{\pi}U_n^\dagger\left[\frac{n}{\pi}\mathcal{B}_0\tilde{c}\left(-\frac{\pi(n-2)}{2n}\right)\tilde{c}\left(\frac{\pi(n-2)}{2n}\right) + \tilde{c}\left(-\frac{\pi(n-2)}{2n}\right) + \tilde{c}\left(\frac{\pi(n-2)}{2n}\right)\right]|0\rangle$$

$$\mathcal{B}_0^\dagger = b_0 + \frac{2}{3}b_{-2} - \frac{2}{15}b_{-4} + \cdots$$

$$U_n^\dagger = \cdots e^{-\frac{16(n^2-4)(n^2-1)(n^2+5)}{945n^6}L_{-8}}e^{\frac{n^4-16}{30n^4}L_{-4}}e^{-\frac{n^2-4}{3n^2}L_{-2}}\left(\frac{2}{n}\right)^{L_0}$$

For $\lambda = 1$, this solution represents tachyon vacuum.

For $0 \leq \lambda < 1$, it corresponds to a pure gauge configuration.



3. Virasoro basis

We will find coefficients in the Virasoro basis of ψ_{n-2} .

Up to level 2, we can express ψ_{n-2} using the Fock space representation;

$$\psi_{n-2} = a_t(n)c_1|0\rangle + a_u(n)c_{-1}|0\rangle + a_v(n)L_{-2}^X c_1|0\rangle + a_w(n)b_{-2}c_0c_1|0\rangle + \cdots,$$

$$a_t(n) = -\frac{n}{\pi}\sin^2\left(\frac{\pi}{n}\right)\left(-1 + \frac{n}{2\pi}\sin\left(\frac{2\pi}{n}\right)\right),$$

$$a_u(n) = -\left(\frac{4}{\pi} - \frac{n}{\pi}\sin^2\left(\frac{\pi}{n}\right)\right)\left(-1 + \frac{n}{2\pi}\sin\left(\frac{2\pi}{n}\right)\right),$$

$$a_v(n) = -\left(\frac{4}{3n\pi} - \frac{n}{3\pi}\right)\sin^2\left(\frac{\pi}{n}\right)\left(-1 + \frac{n}{2\pi}\sin\left(\frac{2\pi}{n}\right)\right),$$

$$a_w(n) = -\sin^2\left(\frac{\pi}{n}\right)\left(\frac{8}{3n\pi} - \frac{2n}{3\pi} + \frac{n^2}{3\pi^2}\sin^2\left(\frac{2\pi}{n}\right)\right)$$

From Schnabl's solution, we can find the Fock space representation of the solution:

$$\Psi(\lambda) = t(\lambda)c_1|0\rangle + u(\lambda)c_{-1}|0\rangle + v(\lambda)L_{-2}^X c_1|0\rangle + w(\lambda)b_{-2}c_0c_1|0\rangle + \cdots,$$

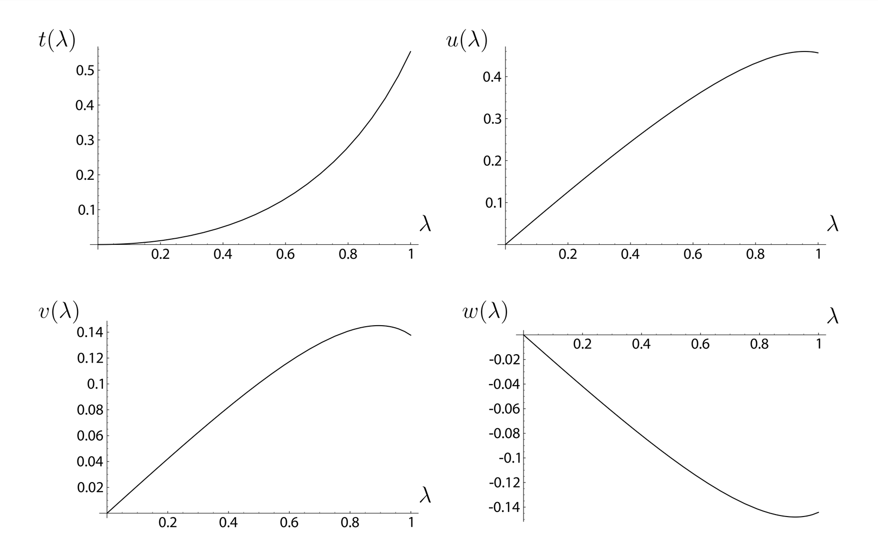
$$t(\lambda) = -\sum_{n=2}^{\infty}\lambda^{n-1}\partial_n a_t(n), \qquad u(\lambda) = -\sum_{n=2}^{\infty}\lambda^{n-1}\partial_n a_u(n),$$

$$v(\lambda) = -\sum_{n=2}^{\infty}\lambda^{n-1}\partial_n a_v(n), \qquad w(\lambda) = -\sum_{n=2}^{\infty}\lambda^{n-1}\partial_n a_w(n)$$

The first term in solution dose not contribute to these coefficients even if the case $\lambda = 1$.

We can evaluate the vacuum energy numerically, in Fig. 1.

Fig. 1



Note that these fields are continuous and dose not jump at $\lambda = 1$.

4. Vacuum Energy

The vacuum energy is given by $V(\lambda) = 2\pi^2\left(\frac{1}{2}\langle\Psi(\lambda), Q_B\Psi(\lambda)\rangle + \frac{1}{3}\langle\Psi(\lambda), \Psi(\lambda) * \Psi(\lambda)\rangle\right)$, including interaction terms. Using the equation of motion, it can be given by evaluating the kinetic term only; $V(\lambda) = \frac{\pi^2}{3}\langle\Psi(\lambda), Q_B\Psi(\lambda)\rangle$.

We expect that as the truncation level is increased the plot of the vacuum energy approaches a step function, *i.e.*

$$V(\lambda) = \begin{cases} 0 & (0 \leq \lambda < 1) \\ -1 & (\lambda = 1) \end{cases}$$

We evaluate the tachyon vacuum energy $(\lambda = 1)$ from the kinetic term and including interaction terms up to level 6.

	$L=2$	$L=4$	$L=6$
kinetic terms only	-1.007815	-1.004499	-1.003217
	level (2,4)	level (4,12)	level (6,18)
incl. interaction terms	-1.006518	-1.004798	-1.003287

The resulting plots approach the plots expected from physical interpretation of the Schnabl's solution as the level is increased.

The plots including the interaction term are slightly changed from the plot via the kinetic term only.

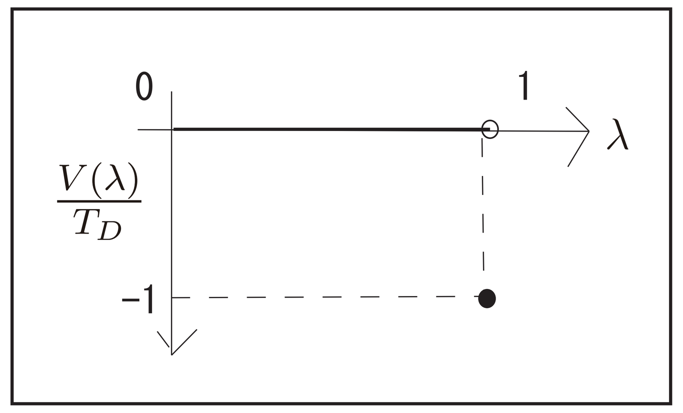


Fig. 2 expected plots for the vacuum energy

Up to level 6

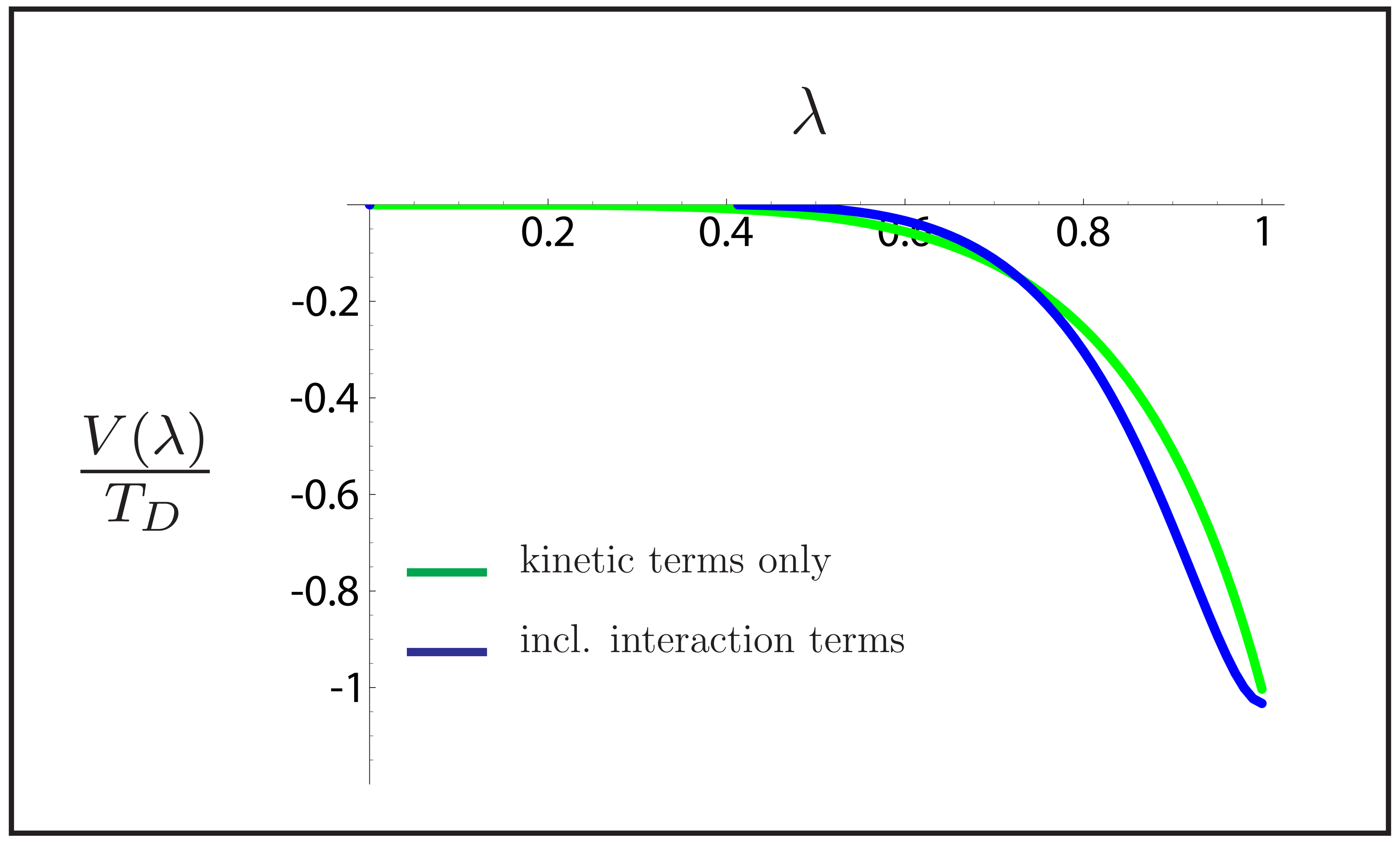


Fig. 3. Vacuum energy of the classical solution $\Psi(\lambda)$ against the parameter λ .

5. Discussion

These results suggest that the Schnabl's solution is well behaved in the standard Fock basis, even for the pure gauge case ($0 < \lambda < 1$). The vacuum energy via the L_0 level truncation seems to approach the correct value as the level is increased, even if the interaction terms are included in the calculation.

The term $\lim_{N\to\infty}\psi_N$ is unnecessary for the correct vacuum energy to be reproduced in the L_0 level truncation, although this term is important difference between the tachyon and pure gauge solutions in the analytic expression using wedge states.

We need further analysis (e.g. higher level truncated calculation) to confirm these results and the fact that the vacuum energy absolutely converges in the L_0 truncation and jumps from zero to minus one at $\lambda = 1$.