A High Statistics Study of the Decay $\tau^- \to \pi^- \pi^0 \nu_\tau$

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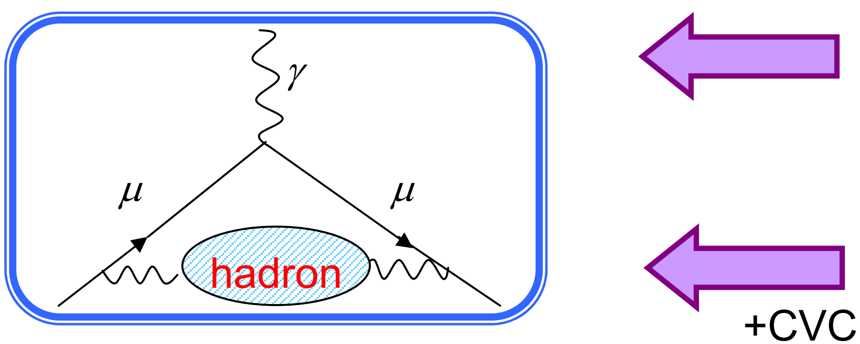
for
the Belle Collaboration

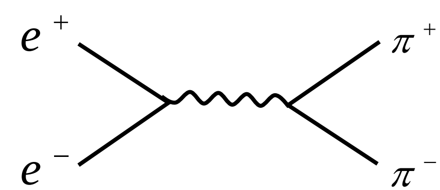
Outline

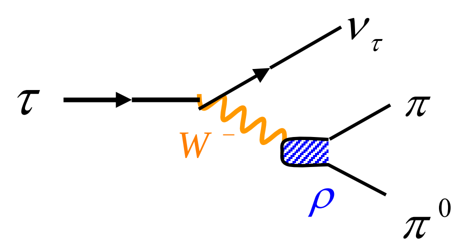
1. Introduction
2. Experiment
   -KEKB and Belle-
3. Analysis
   Event selection
   Mass spectrum
   Muon anomalous magnetic moment
4. Summary

Introduction

- The hadronic vacuum polarization term plays an important role in the theoretical calculation of the muon anomalous magnetic moment.
- The dominant part of the hadronic vacuum polarization term can be calculated from the 2π Spectral function measured with e^+e^- or τ data.
- Recent data indicate that there is a systematic difference between the 2π system in e^+e^- reaction and τ-decays, which needs to be understood.







- In this talk, results from Belle experiment are presented.
  - Results are based on 72.2/fb data taken in 2000-2002 period.
  - One order of magnitude bigger than preceding experiments.

What should be measured

$$a_\mu^{\pi\pi} = \frac{\alpha(0)^2}{\pi}\int_{4m_\pi^2}^{\infty} ds \frac{K(s)}{s} v^{\pi\pi}(s) \longleftarrow \text{spectral function}$$

$$\begin{cases} \alpha : \text{fine structure constant} \\ s = M_{\pi\pi^0}^2 \\ K(s) = x^2\left(1-\frac{x^2}{2}\right)+(1+x)^2\left(1+\frac{1}{x^2}\right)\left(\ln(1+x)-x-\frac{x^2}{2}\right)+\left(\frac{1+x}{1-x}\right)x^2\ln x \\ x = \frac{1-\beta_\mu}{1+\beta_\mu} \quad , \quad \beta_\mu = \sqrt{1-4m_\mu^2/s} \end{cases}$$

$$v^{\pi\pi}(s) = \frac{m_{\tau^2}}{6\pi|V_{ud}|^2 S_{EW}} \cdot \frac{B_{\pi\pi^0}}{B_e} \cdot \left[\left(1-\frac{s}{m_{\tau^2}}\right)^2\left(1+\frac{2s}{m_{\tau^2}}\right)\right]^{-1} \cdot \frac{1}{N_{\pi\pi^0}}\frac{dN_{\pi\pi^0}}{ds}$$

- Branching Fraction
- Mass Spectrum

Experiment apparatus :

Belle detector

SC solenoid 1.5T

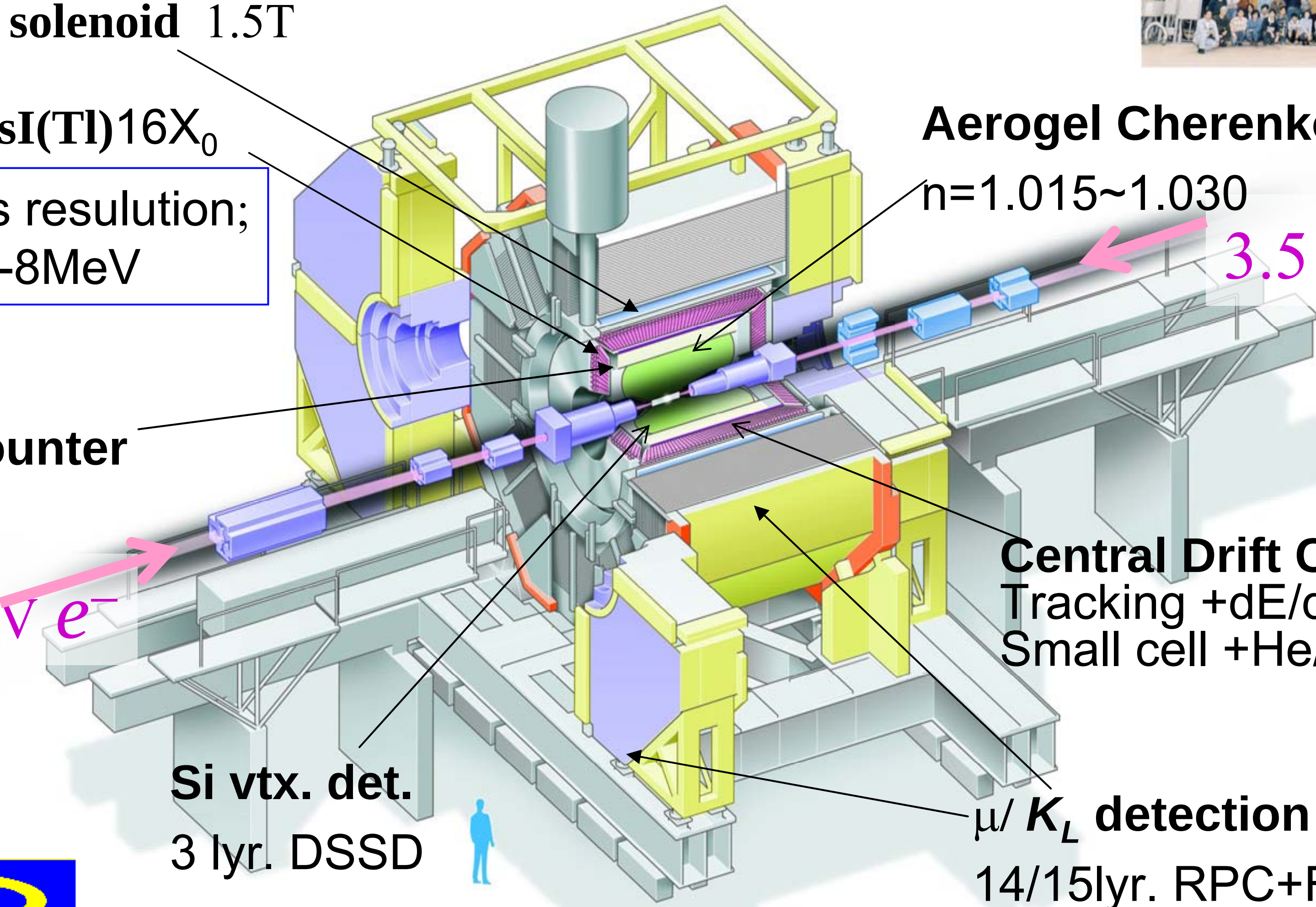
CsI(Tl) $16X_0$

π^0 mass resulution;
$\sigma_{\pi^0} \sim$ 5 -8MeV

Aerogel Cherenkov cnt.
n=1.015~1.030

3.5 GeV e^+

TOF counter



8 GeV e^-

Central Drift Chamber
Tracking +dE/dx
Small cell +He/C_2H_6

Si vtx. det.
3 lyr. DSSD

μ/ K_L detection
14/15lyr. RPC+Fe

BELLE

Good tracking and particle identification

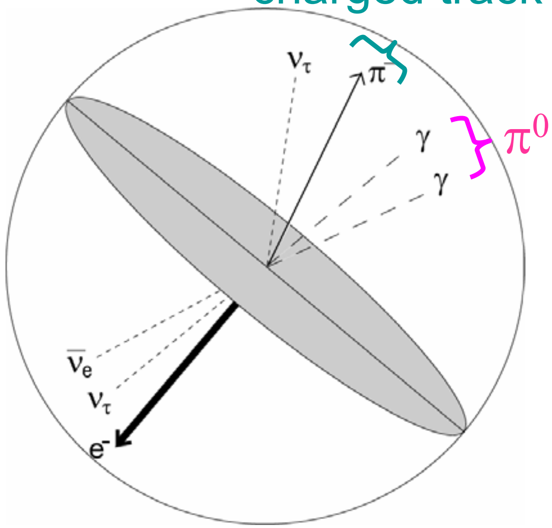
Event Selection

$e^+e^- \rightarrow \tau^+\tau^-$ Selection

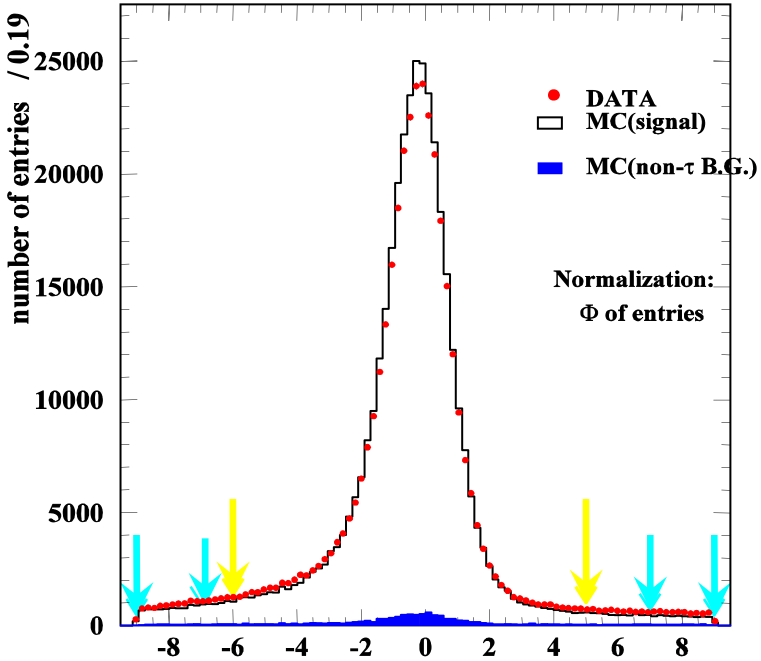
- Low multiplicity:
  Number of charged tracks : 2 or 4, net charge=0
- Beam background rejection: Event Vertex Position
- Physics background rejection:
  - Use Missing Mass and Missing Angle information.(Bhabha,2photon)
  - Low track and gamma multiplicity. (qq continuum)

$\tau^- \rightarrow \pi^-\pi^0\nu_\tau$ Selection

- one charged track in the event hemisphere.
- one π^0 in the event hemisphere.
- No additional γ with $E_\gamma \geq 200$MeV
- Tag-side condition
  Tag-side : 1 prong and no γ
  (← reduce continuum B.G. at $m_{\pi\pi^0} \sim m_\tau$)



π^0 Signal



$$S_{\gamma\gamma} \equiv \frac{(m_{\gamma\gamma} - m_{\pi^0})}{\sigma_{\gamma\gamma}}$$

m_{π^0} : π^0 **Mass** (134.98_{MeV})

$m_{\gamma\gamma}$: $\gamma\gamma$ **invariant mass distribution**

$\sigma_{\gamma\gamma}$: **resolution of** $m_{\gamma\gamma}$

Signal region

$-6 < S_{\gamma\gamma} < 5$

Sideband region

right: $-9 < S_{\gamma\gamma} < -7$

left: $7 < S_{\gamma\gamma} < 9$

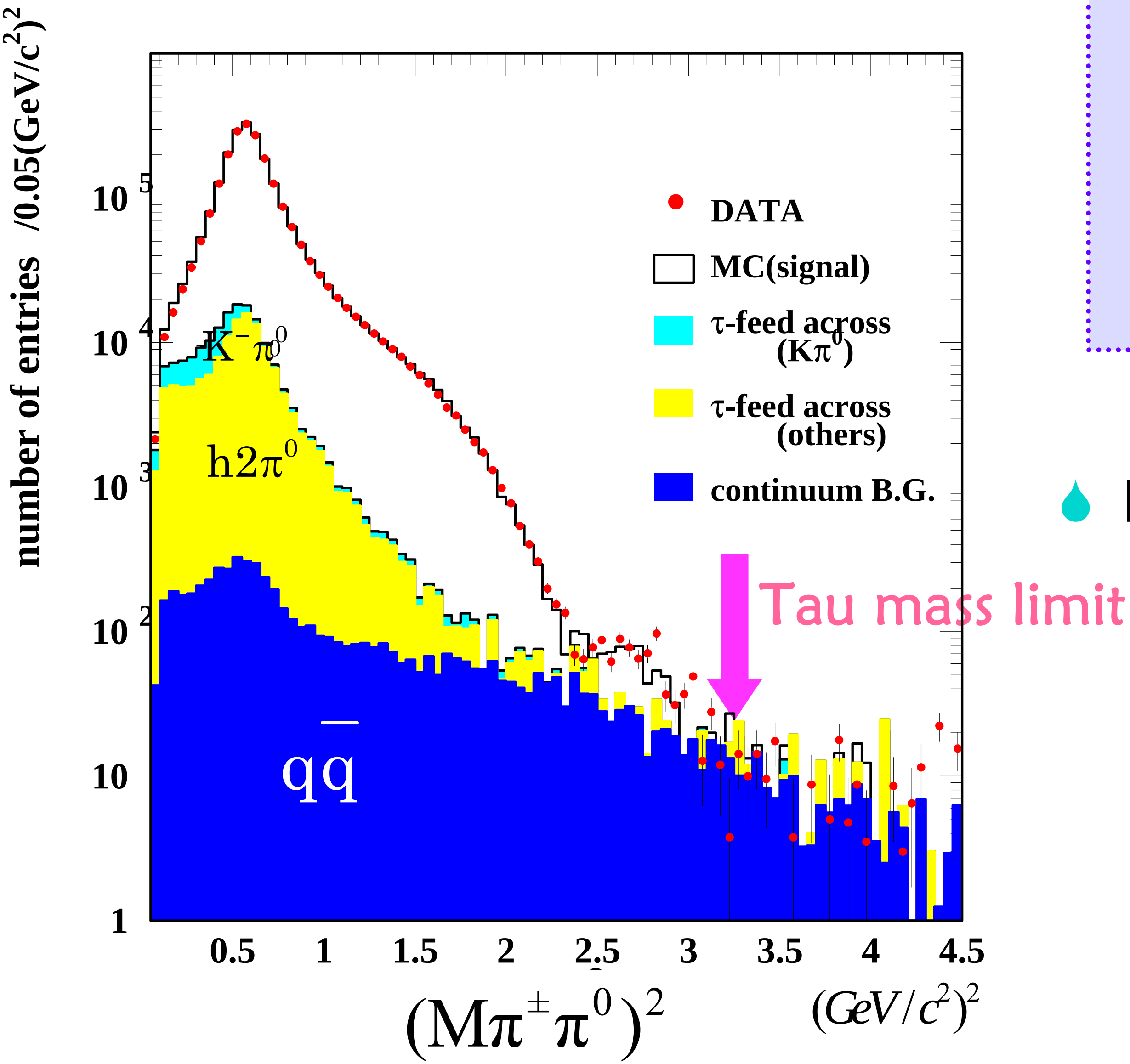
Sideband region is used to estimate the non-π^0 background

72.2/fb data at $\sqrt{s_{ee}} = 10.58\text{GeV}$

$\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$

5.55×10^6 event

$m^2_{\pi\pi}$ distribution



tag side

1 charged track
+
no γ

- B.G. estimated by MC
  - non-τ B.G.
    - $q\bar{q}$ $0.25 \pm 0.02\%$
  - feed across B.G.
    - $h \geq 2\pi^0\nu_\tau$ 5.87%
    - $K^-\pi^0\nu_\tau$ 1.76%

Analysis procedure

$m^2_{\pi\pi}$ distribution

↓

Background subtraction

↓

Acceptance correction(Unfolding)

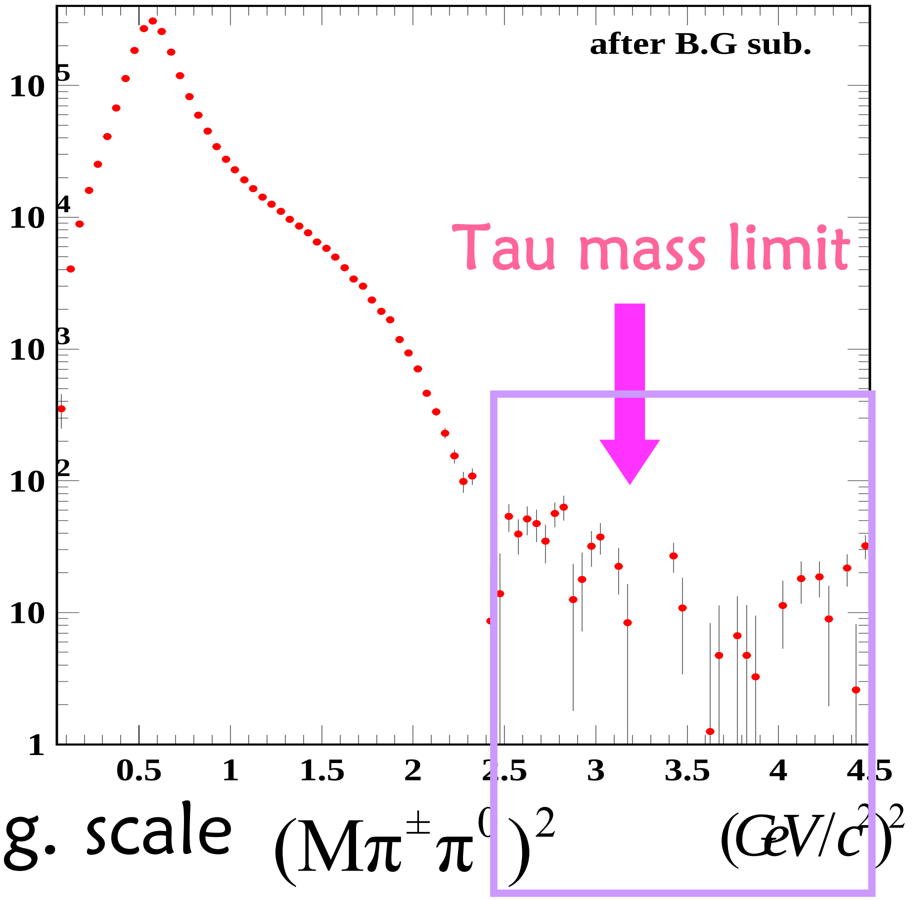
↓

Radiative(ISR) correction

↓

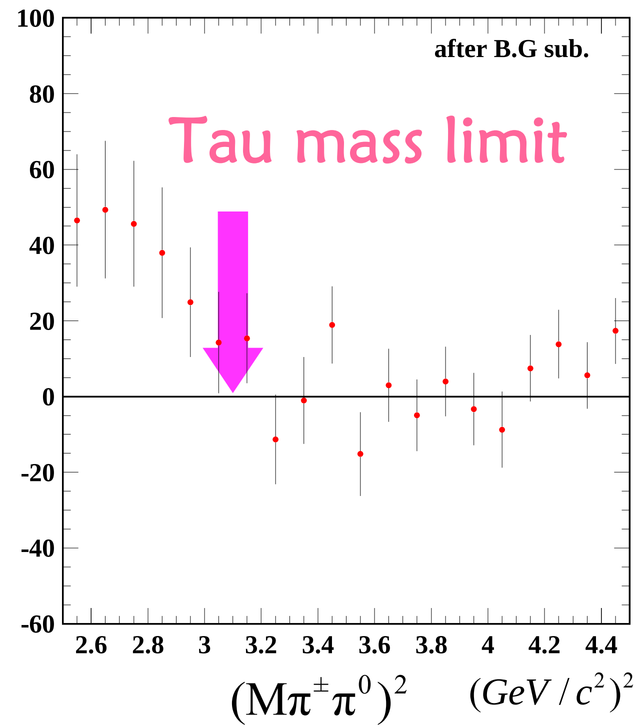
Mass spectrum after Unfolding

B.G. subtracted $m^2_{\pi\pi}$ distribution



Log. scale

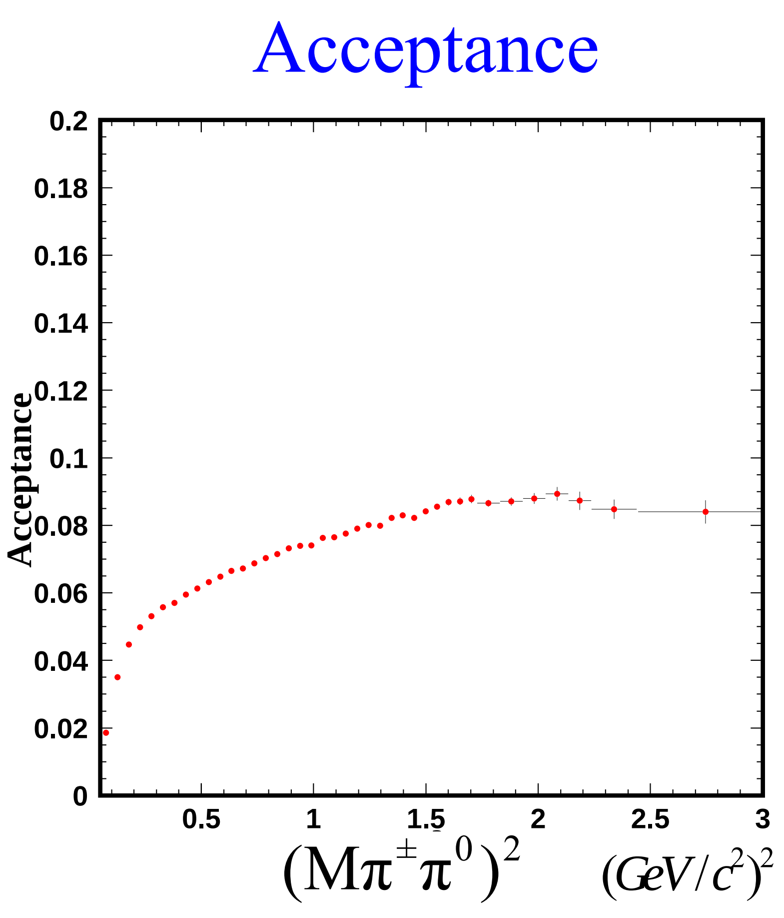
Linear scale

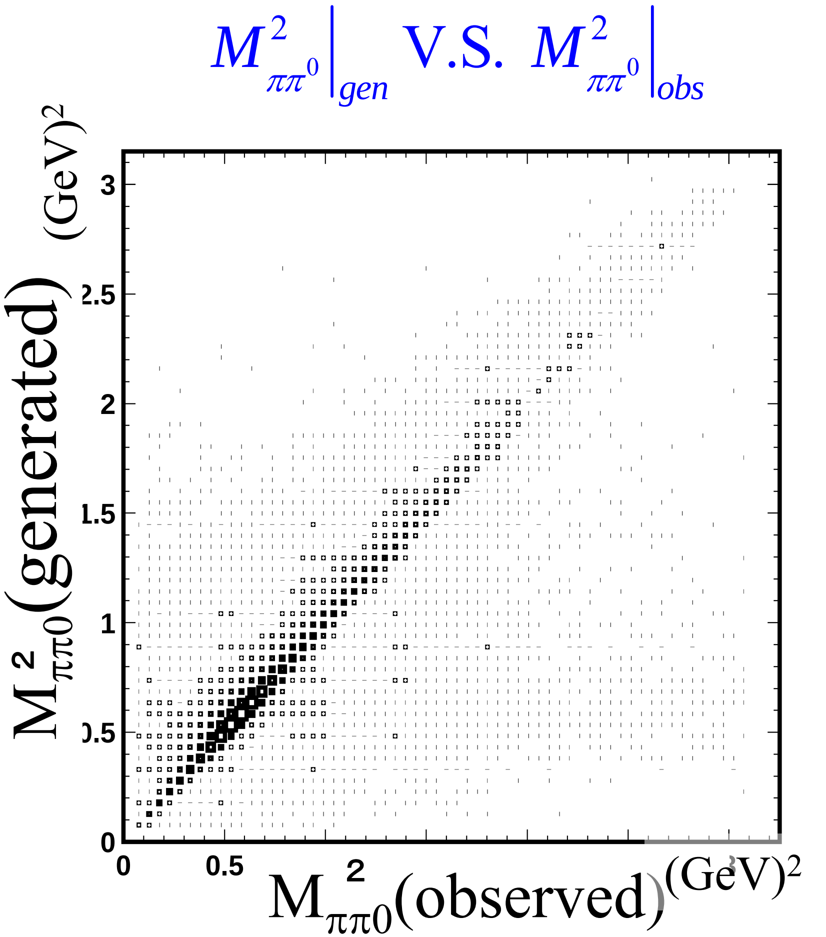


- 2bins are combined
- $N(m^2_{\pi\pi} > m^2_{\tau}) = (2.3 \pm 7.8)$ events

Acceptance

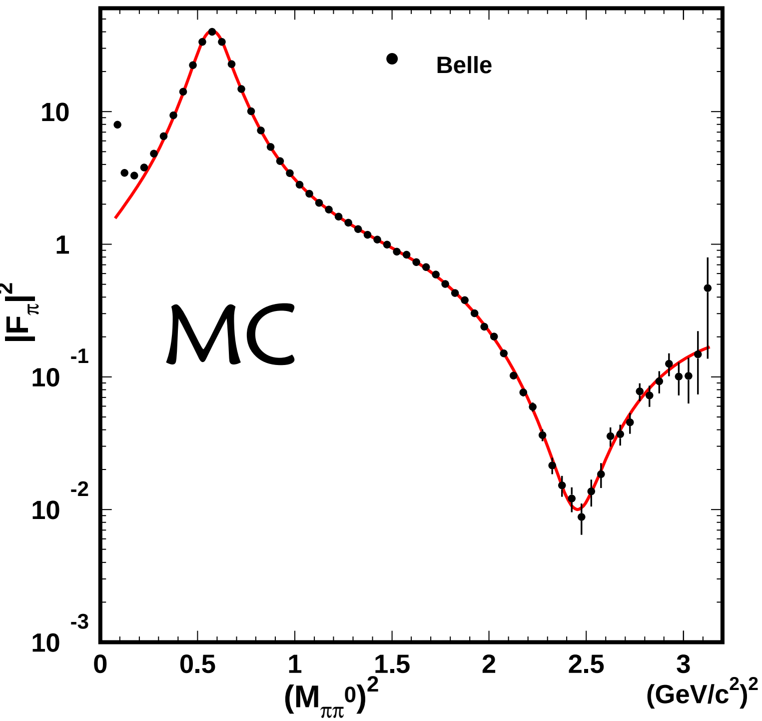
Data are Unfolded with
the Singular Value Decomposition (SVD) method.





Radiative correction

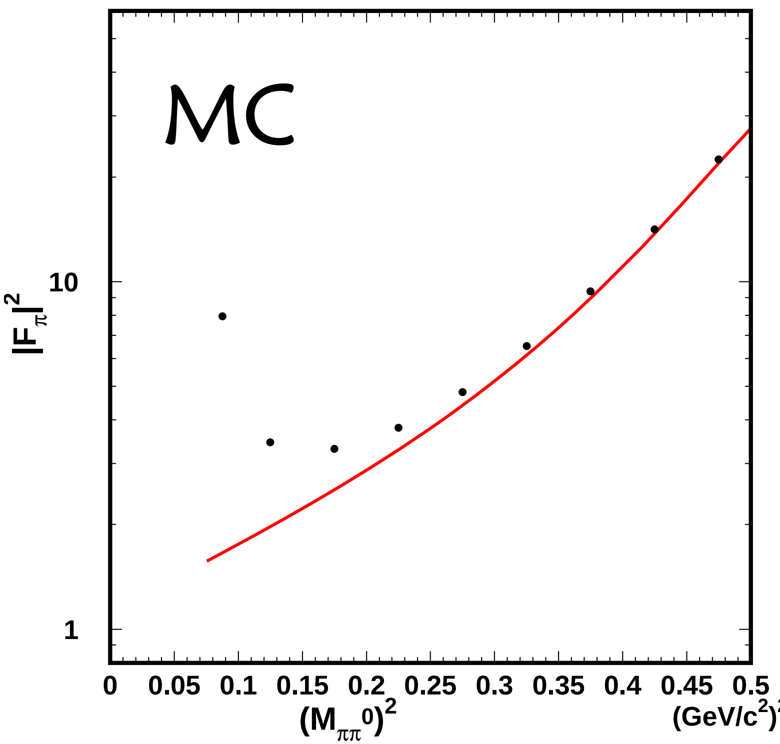
- Generator level distribution with MC



Line : Born level distribution

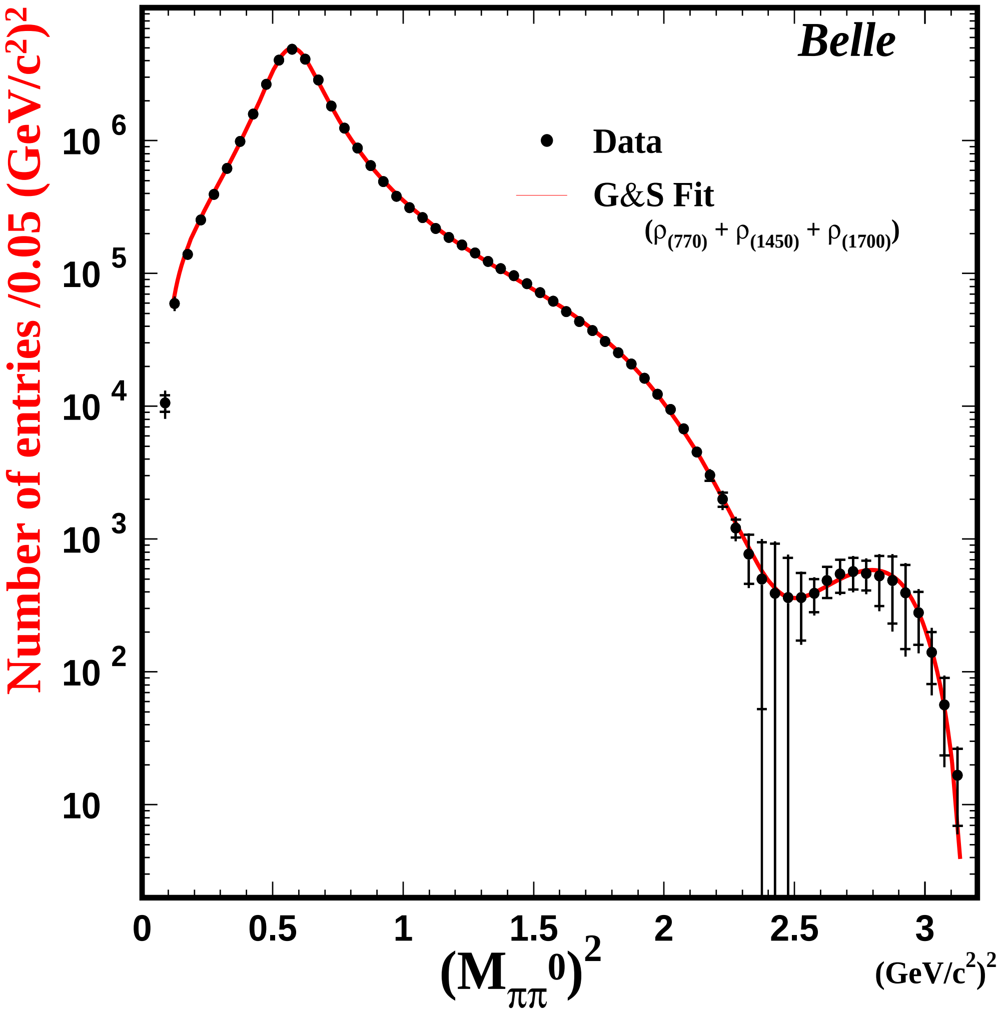
Plot : KKMC/Tauola

(w/ radiative correction)



This effect is corrected in data

Mass spectrum after Unfolding



Mass spectra

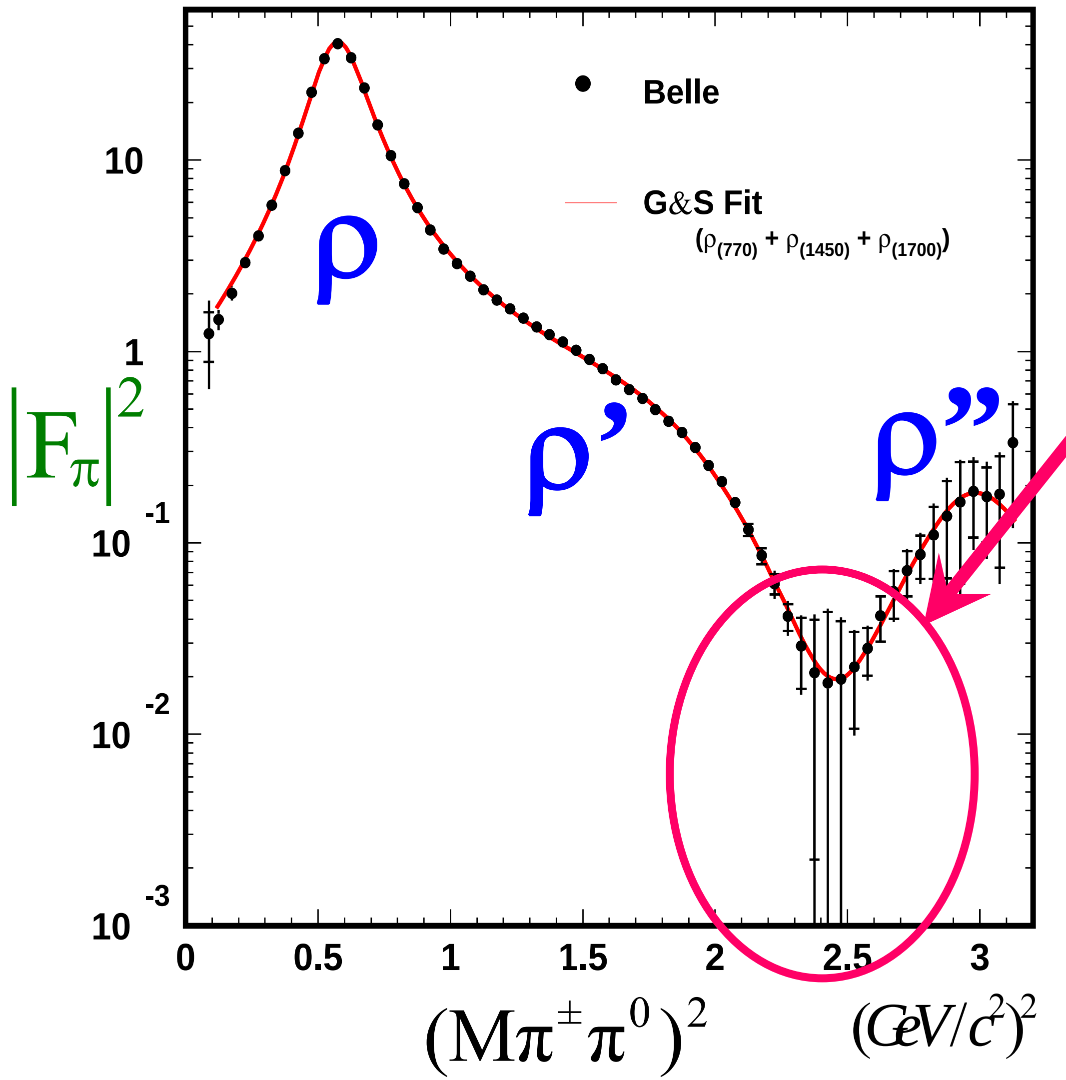
= Phase space

× Form Factor

$$\frac{1}{N_{\pi\pi^0}}\frac{dN_{\pi\pi^0}}{ds} = \frac{6\pi|V_{ud}|^2 S_{EW}}{m_\tau^2} \times \frac{B_e}{B_{\pi\pi}}\left[\left(1-\frac{s}{m_\tau^2}\right)^2\left(1+\frac{2s}{m_\tau^2}\right)\right] v^{\pi\pi^0}(s)$$

$$v^{\pi\pi^0}(s) = \frac{\beta^3(s)}{12\pi}\left|F_\pi(s)\right|^2$$

Pion Form Factor $|F_\pi|^2$



- Fit with BW Forms. →Next slide
- Interference between ρ' and ρ''

Dip at s=2.5 GeV2

Systematic in the mass distribution
checked items

- Unfolding procedure
  - Checked with Signal MC（UNF①）
  - Unfolding condition : value±5 （UNF②）
- Background (BKG)
  - Main contribution is continuum
    - Continuum ±10%
    - Checked by Continuum Enhanced sample
- Acceptance (Accep.)
  - Pi0 efficiency ±3%
  - Gamma track isolation effect
    - Change Track – cluster distance cut（default , tighter30cm）
- Momentum or energy scale (ENS)
  - Change $E\gamma$ ±0.2%

Systematic of mass squared distribution

$M^2_{\pi\pi}$	1^{st} bin $0.08GeV^2$	threshold $0.2\text{-}0.3GeV^2$	ρ region	ρ' region	ρ" region
UNF①	**0.0546**	**0.0069**	**0.0004**	**0.0232**	**0.0871**
UNF②	**0.0263**	**0.0019**	**0.0022**	**0.0037**	**0.1693**
BKG	**0.0113**	**0.0007**	**0.0002**	**0.0008**	**0.0488**
Accep.	**0.0536**	**0.0004**	**0.0032**	**0.0016**	**0.0005**
ENS	**0.0124**	**0.0036**	**0.0031**	**0.0408**	**0.0167**
Total	**0.082**	**0.008**	**0.005**	**0.005**	**0.197**

- A relative systematic $(N - N_{ref})/ N_{ref}$ shown for each mass region

Fitting by BW formula

ρ(770), ρ'(1400), ρ''(1700)

$$F_\pi(s) = \frac{1}{1+\beta+\gamma}(BW_\rho + \beta BW_{\rho'} + \gamma BW_{\rho''})$$

$$BW_\rho^{G\&S} = \frac{M_\rho^2 + d(s) M_\rho \Gamma_\rho(s)}{(M_\rho^2 - s) + f(s) - i\sqrt{s}\Gamma_\rho(s)}$$

Gounaris-Sakurai(GS) parameterization

fit parameter

M_ρ , Γ_ρ	: ρ mass and width
$M_{\rho'}$, $\Gamma_{\rho'}$	: ρ′ mass and width
$M_{\rho''}$, $\Gamma_{\rho''}$	: ρ″ mass and width
β , ϕ_β	: ρ′ amplitude
γ , ϕ_γ	: ρ″ amplitude

The normalization of the GS form is given by $|F_\pi(0)|^2 = 1$

Two kinds of fits applied:

- Fixed $|F_\pi(0)|^2 = 1$
- Make $|F_\pi(0)|^2$ as a free parameter

Fit result

fit parameter	all free	Norm fixed
Norm $\lvert F_\pi(0) \rvert^2$	1.06±0.01	**1.0 (fixed)**
M_ρ (MeV)	774.2±0.3	**773.5±0.2±0.7**
Γ_ρ (MeV)	149.4±0.4	**149.2±0.4±0.8**
$M_{\rho'}$ (MeV)	1424±12	**1453±7±29**
$\Gamma_{\rho'}$ (MeV)	479.9±22.9	**437.6±19.9±80**
$\lvert \beta \rvert$	0.136±0.010	**0.167±0.005±0.046**
ϕ_β (degree)	175.7±7.1	**210.3±6.3±40**
$M_{\rho''}$ (MeV)	1688±16	**1730±22±113**
$\Gamma_{\rho''}$ (MeV)	244.3±25.8	**137.9±50.0±88**
$\lvert \gamma \rvert$	0.061±0.009	**0.031±0.011±0.05**
ϕ_γ (degree)	-16.3±7.6	**44.2±17±117**
$\chi^2 / d.o.f$	80/51	**91/52**

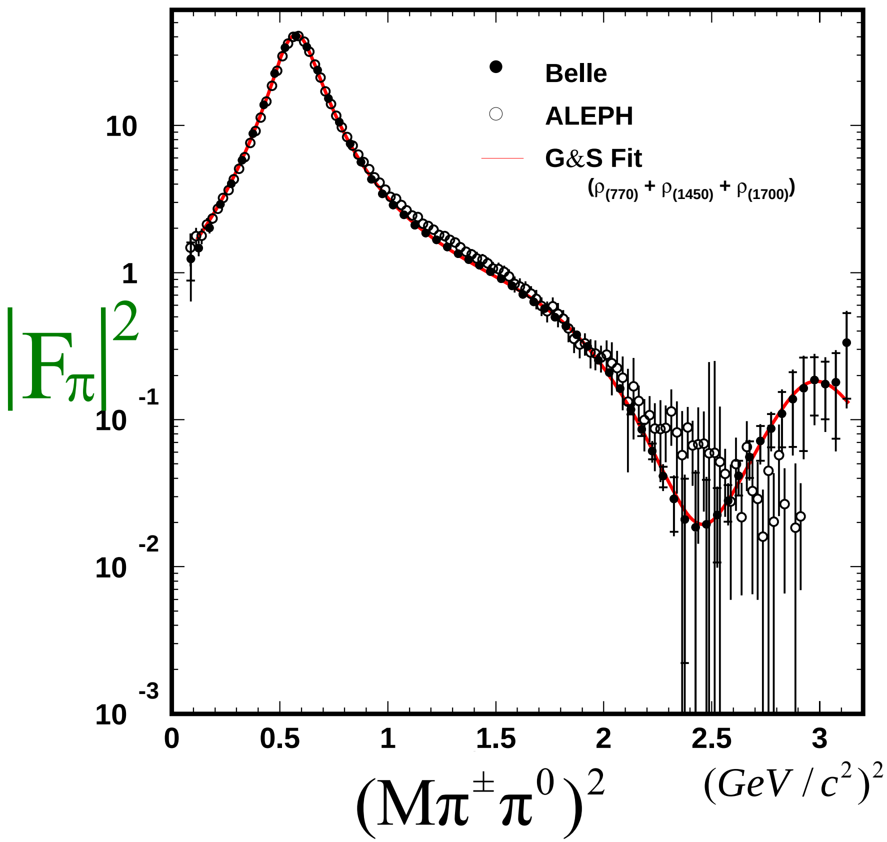
Systematic of resonance parameters

Source of systematics	$M\rho$ (MeV)	$\Gamma\rho$ (MeV)	$M\rho'$ (MeV)	$\Gamma\rho'$ (MeV)	β	ϕ_β (deg.)	$M\rho''$ (MeV)	$\Gamma\rho''$ (MeV)	γ	ϕ_γ (deg.)
Fit bias	**0.5**	**0.4**	**27**	**71**	**0.037**	**4**	**103**	**	**	**
unfold	**0.3**	**0.3**	**3**	**26**	**0.02**	**0.1**	**11**	**7.2**	**0.002**	**6**
B.G.	**0.2**	**--**	**11**	**25**	**0.014**	**40**	**13**	**86**	**0.053**	**117**
Acceptance	**--**	**0.1**	**1**	**4**	**---**	**0.6**	**0.1**	**7**	**---**	**1**
Momentum scale	**0.3**	**0.6**	**2**	**1**	**---**	**2**	**45**	**15**	**---**	**1**
total	**0.7**	**0.8**	**29**	**80**	**0.046**	**40**	**113**	**88**	**0.05**	**117**

"Fit bias" is checked by fitting the signal MC sample, where resonance parameter is known.

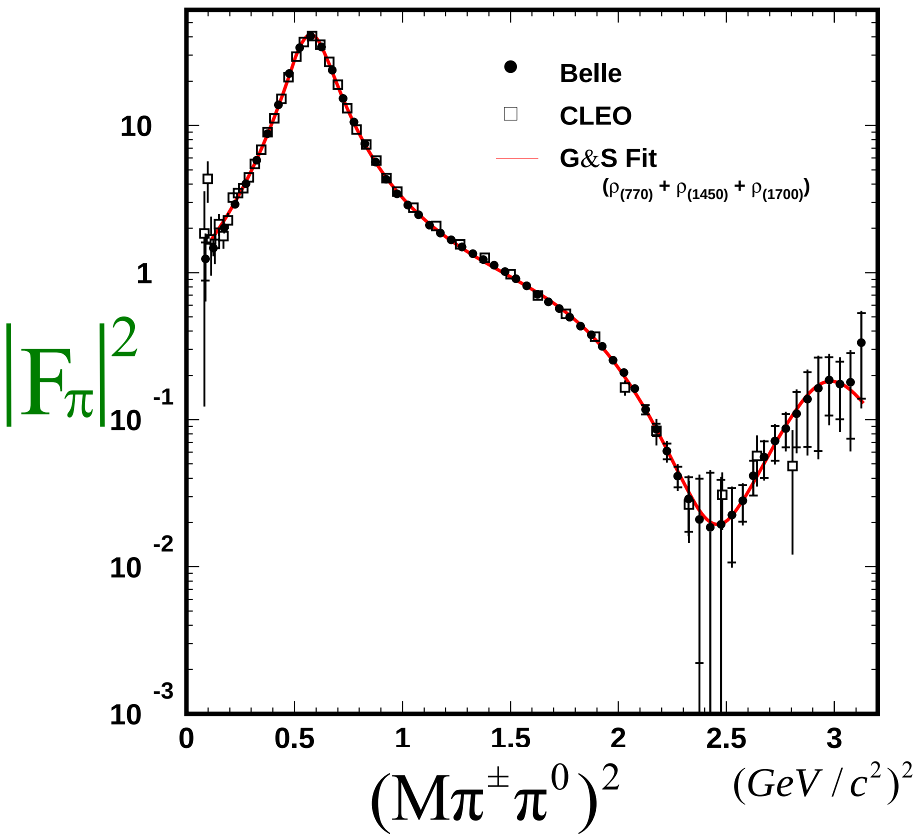
Comparison with ALEPH,CLEO

BELLE & ALEPH



Ref: Phys. Rep.421 (2005) 191

BELLE & CLEO

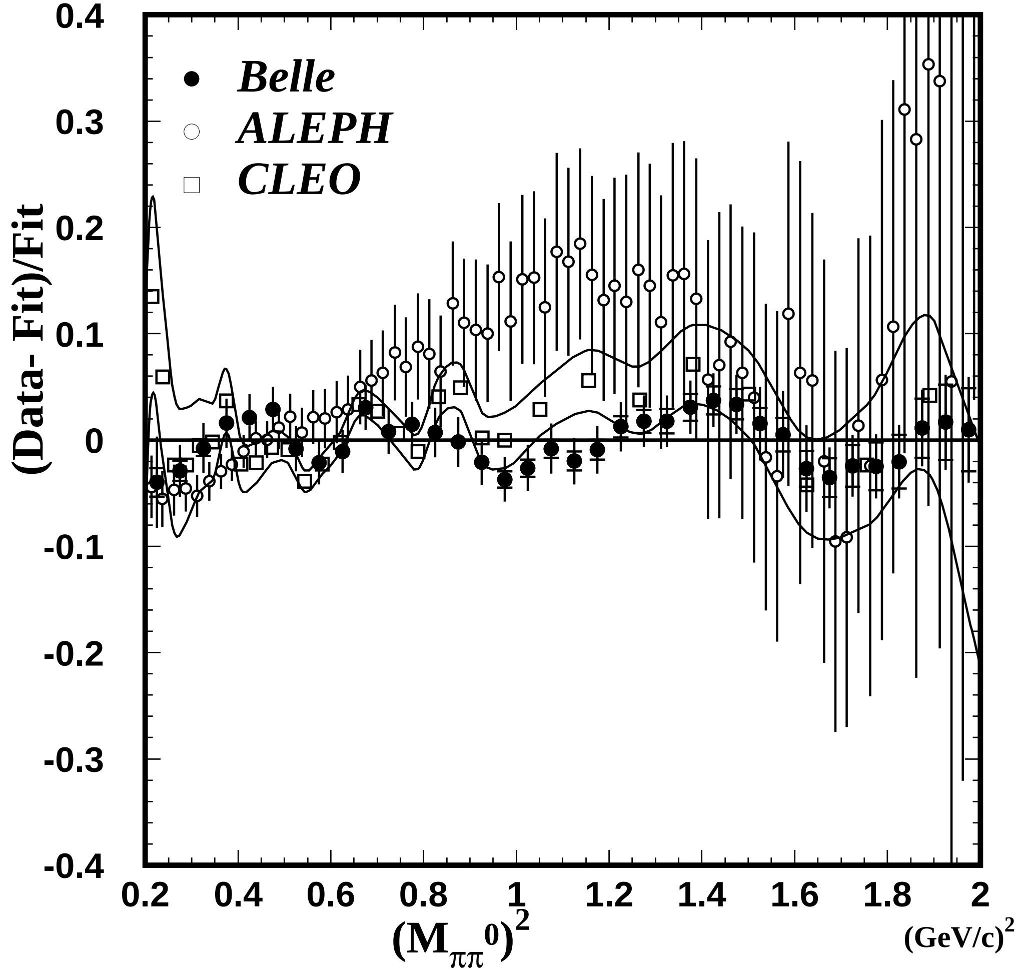


Ref: Phys. Rev. D61, 112002(2000) 1

Agree with other exp. data.

Our result is more precise especially in high mass region.

Ratio of data/fit



Hadronic vacuum polarization a_μ

$$a_\mu^{had,LO} = \frac{\alpha^2}{\pi} \int_{4m_\pi^2}^{\infty} ds \frac{K(s)}{s} \; v^{\pi\pi}(s)$$

$$v^{\pi\pi}(s) = \frac{m_{\tau^2}}{6\pi |V_{ud}|^2 S_{EW}} \cdot \frac{B_{\pi\pi^0}}{B_e} \cdot \left[\left(1 - \frac{s}{m_{\tau^2}}\right)^2 \left(1 + \frac{2s}{m_{\tau^2}}\right) \right]^{-1} \cdot \frac{1}{N_{\pi\pi^0}} \frac{dN_{\pi\pi^0}}{ds}$$

$$a_\mu(2\pi) = 461.6 \pm 0.5(\text{stat.}) \pm 1.0(\text{int.sys.}) \pm \underline{3.0(\text{ext.sys})} \times 10^{-10}$$

$$0.25\,\text{GeV}^2 \leq m_{\pi\pi}^2 \leq m_\tau^2$$

$a_\mu(2\pi)$ after ~~SU(2)~~ correction

- Isospin breaking correction in this mass region.

$(-1.8 \pm 2.3)\text{x}10^{-10}$

Ref.Phys.Lett.B513,361(2001)

- ρ-ω interference effects
- $m_{\pi^\pm} \neq m_{\pi^0}$ in the phase space
- $m_{\pi^\pm} \neq m_{\pi^0}$ in the width

$$a_\mu(2\pi) = 459.8 \pm 0.5(\text{stat.}) \pm 3.2(\text{sys.}) \pm 2.3(\text{SU(2)})$$

$$0.25\text{GeV}^2 \leq m_{\pi\pi}^2 \leq m_\tau^2 \qquad \times 10^{-10}$$

- Consistent with other τ data

c.f. $\tau\ (\text{ALEPH, CLEO})$

$$a_\mu(2\pi) = 464.0 \pm 3.2 \pm 2.3_{\text{SU(2)}}$$

Ref. Eur.Phys.C27,497(2003)

$e^+e^-(\text{CMD2} + \text{KLOE})$

$$a_\mu(2\pi) = 450.2 \pm 4.9 \pm 1.6_{\text{rad}}$$

Ref. Eur.Phys.C31,503(2003)

contribution from each mass region

$$a_\mu^{had,LO} = \frac{\alpha^2}{\pi}\int_{4m_\pi^2}^{\infty} ds \frac{K(s)}{s} \; v^{\pi\pi}(s)$$

$\times 10^{-10}$

$M^2_{\pi\pi}$ region (GeV^2)	Belle	CLEO	ALEPH
0.25-0.45	119.6±0.4	123.6±1.7	113.8±3.5
0.45-0.75	302.7±0.3	298.5±1.4	296.7±2.6
0.75-1.1	32.5±0.1	29.1±0.3	34.4 ±0.7
1.1-1.7	6.1±0.02	6.2 ±0.1	6.9 ±0.2
1.7-3.2	0.81±0.01	0.72 ±0.03	0.78 ± 0.05

Summary

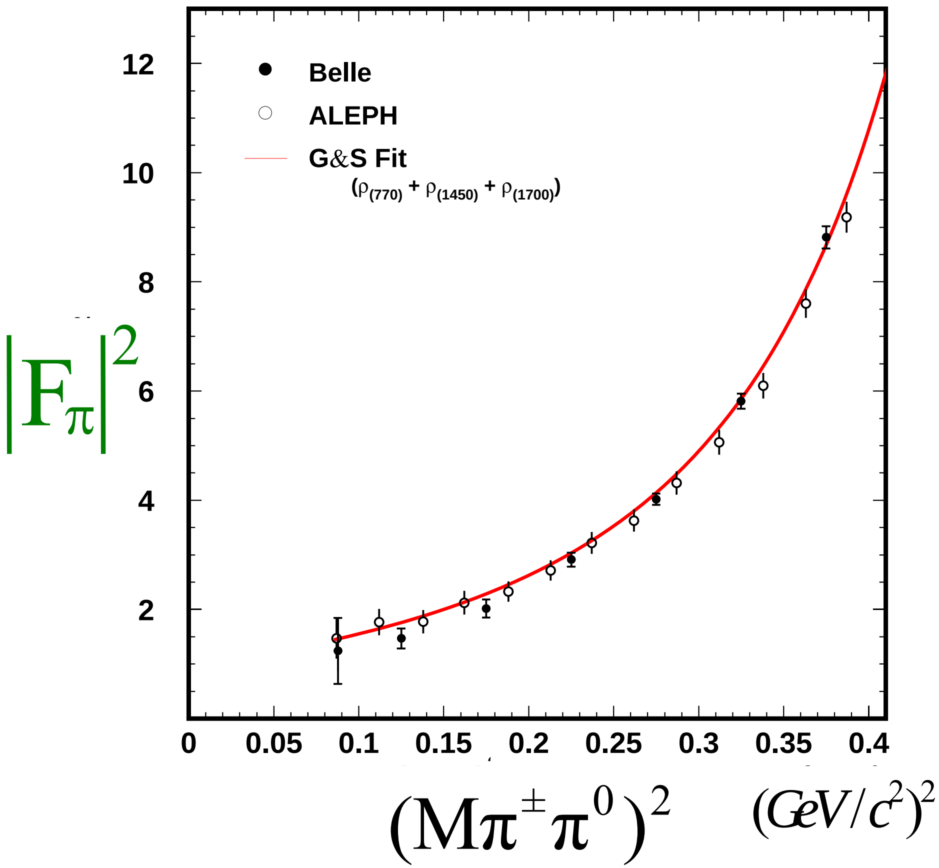
- We measure $\pi^-\pi^0$mass spectrum in the $\tau^- \rightarrow \pi^-\pi^0\nu_\tau$ decay with 72.2/fb data collected with the Belle detector at KEKB.
- In addition to the ρ(770) and ρ'(1400) , the production of the ρ"(1700) in τ decays is unambiguously observed and its parameters are determined.
- We evaluate the 2π contribution to the muon anomalous magnetic moment a_μ using mass spectrum.
- Our a_μ result agrees well with the preceding τ-based results.
- The difference between our τ result and e^+e^- results is 1.5σ.

Backup slides

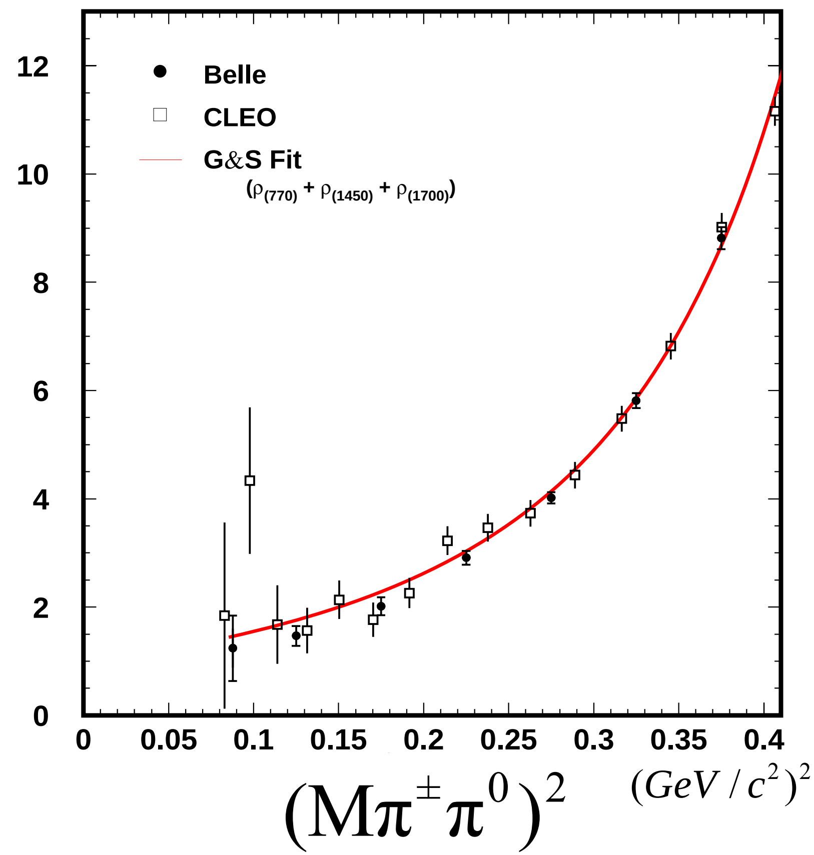
Pion Form Factor $|F_\pi|^2$

-low mass region-

ALEPH



CLEO



Internal Systematic Error

$m_{\pi\pi}^2 \geq 0.25\text{GeV}^2$

source	$\Delta a_\mu^{\pi\pi}$ $(unit : \times 10^{-10})$
Background estimation	
・non- τ (ee->hadron)	±0.11
・feed-down $h \geq 2\pi^0\nu$	±0.09
・feed-down $K^-\pi^0\nu$	±0.15
π^0/γ selection	
efficiency/shape cuts	±0.35
Energy scale	±0.10
Gamma veto	±0.93
γ/track overlap	0.24
Tagging Dependence	<0.1
Smearing/Migration effect	
Total	±1.04

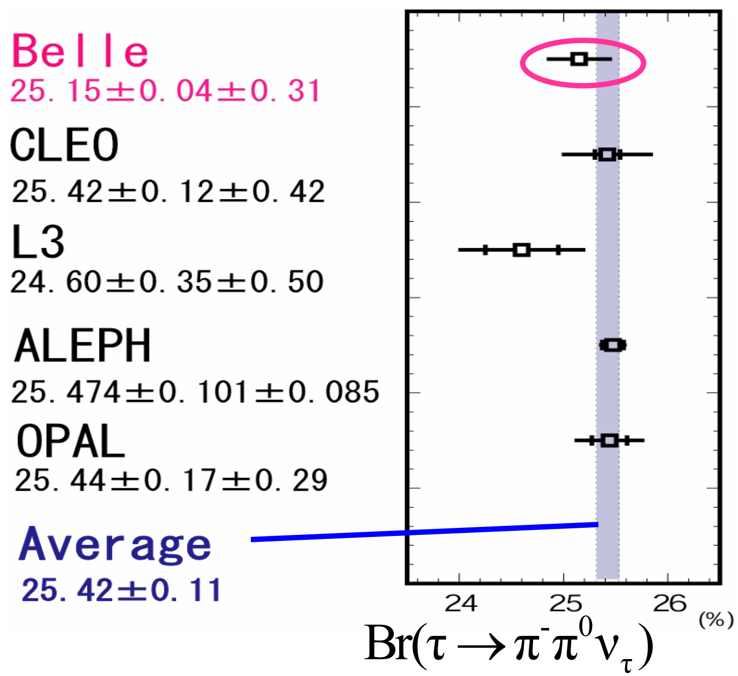
Br pipi0 result:

$$B_{h\pi^0} = (25.60 \pm 0.04(stat) \pm 0.31(sys))\%$$

Subtract kaon branching fraction

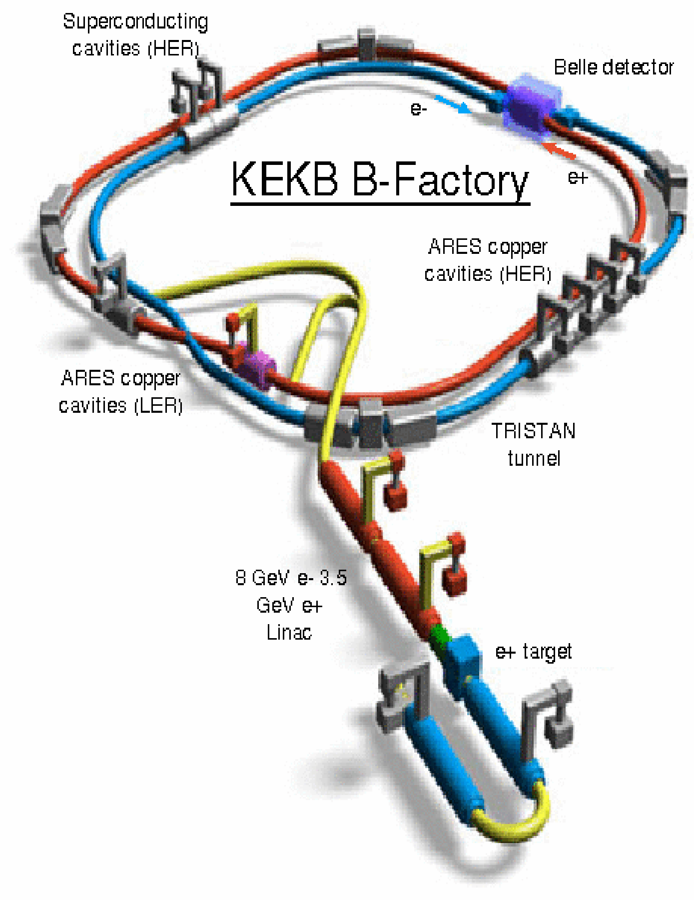
Br(Kpi0) = (0.45 ± 0.03)%

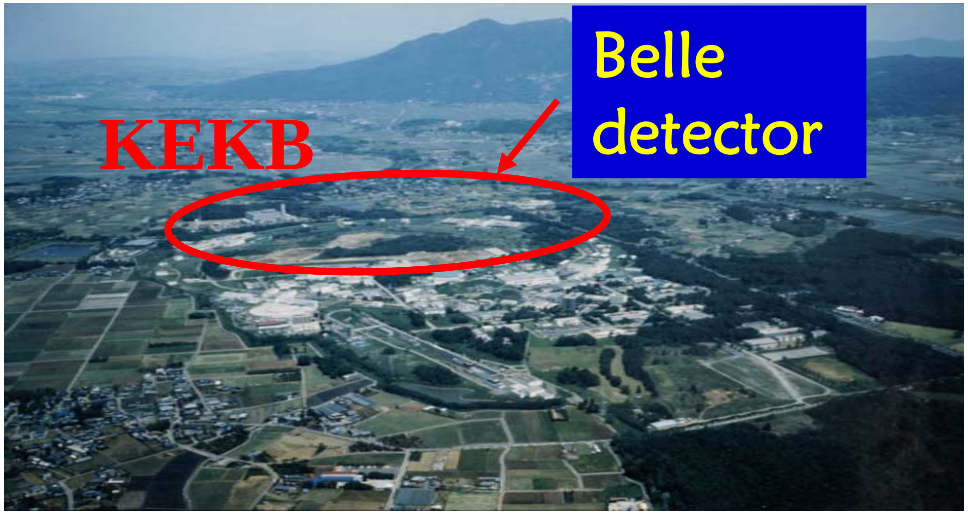
$$B_{\pi\pi^0} = (25.15 \pm 0.04(stat) \pm 0.31(sys))\%$$



Experiment apparatus :

KEKB Collider





- KEKB Collider
  - High Luminosity
  - Asymmetric energy collider
    8GeV :e^- + 3.5GeV:e^+
  - $\sqrt{s}$ = 10.58GeV ($\Upsilon(4S)$)
    $e^+e^- \to \Upsilon(4S) \to B\ \bar{B}$
- **Integrated Luminosity: ~ 630 fb^{-1}**
  ~ 30fb^{-1} => off-resonance

$L > 1.6 \times 10^{34} cm^{-2} s^{-1}$!!