Realizing Topological Chaos by Simple Mechanisms

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joint work with

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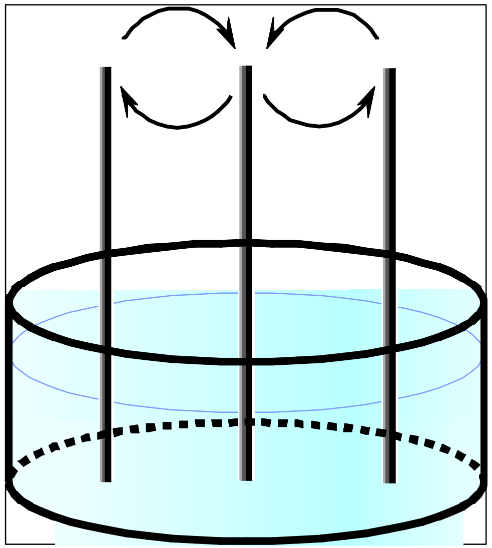
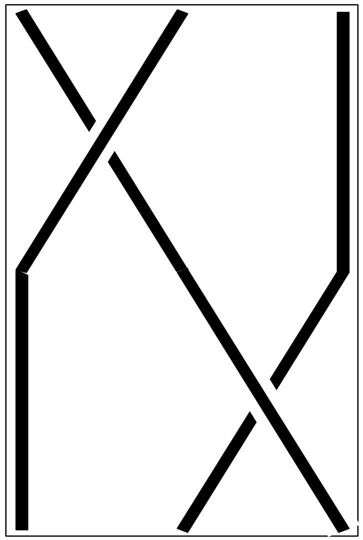
Mixing fluid by a periodic motion of finite number of rods

⇔ Mapping class group of Dn

(: disk with n-punctures)





⇔ Braid group Bn / center

Nielsen-Thurston theory

Each element of Map(Dn) is either
- periodic
- pseudo-Anosov (p.A.) : chaotic
- reducible

Thurston, W., On the geometry and dynamics of diffeomorphisms of surfaces, Bull. Amer. Math. Soc(N. S.) 19(1988), 417-431.

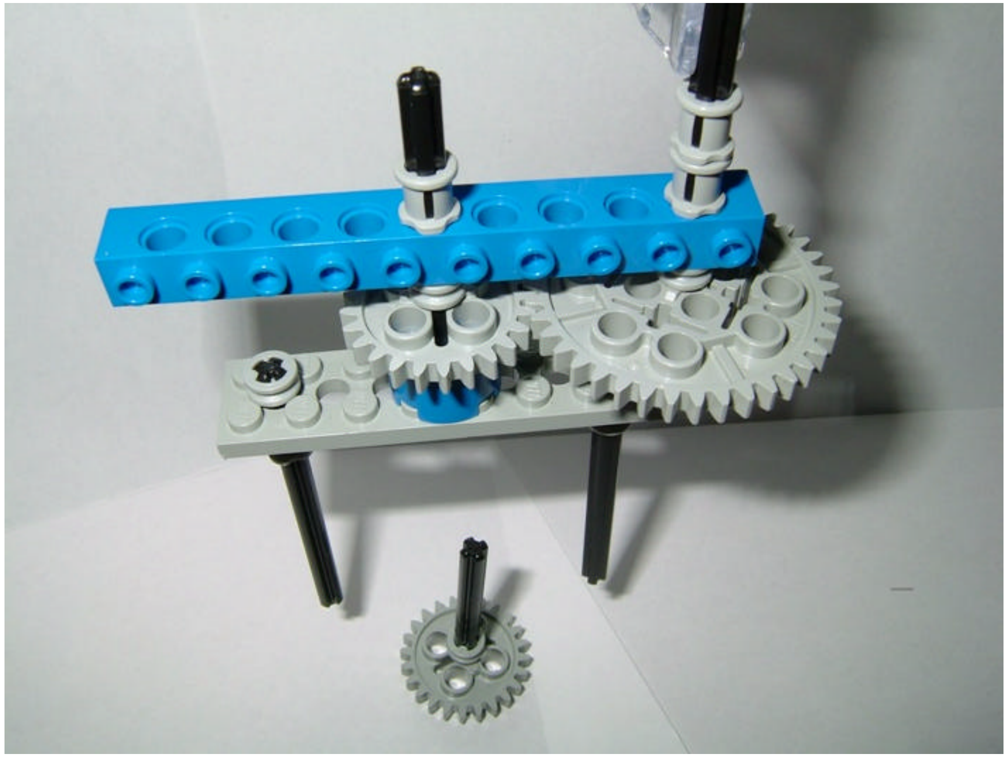
It is natural to expect :

Movement of rods corresponding to p.A. map can mix up fluid efficiently.

$\Rightarrow$B-A-S etc.

Boyland, P.L., Aref, H. , and Stremler M.A., Topological Fluid mechanics of stirring, J.Fluid Mech. 403(2000), 277-304.

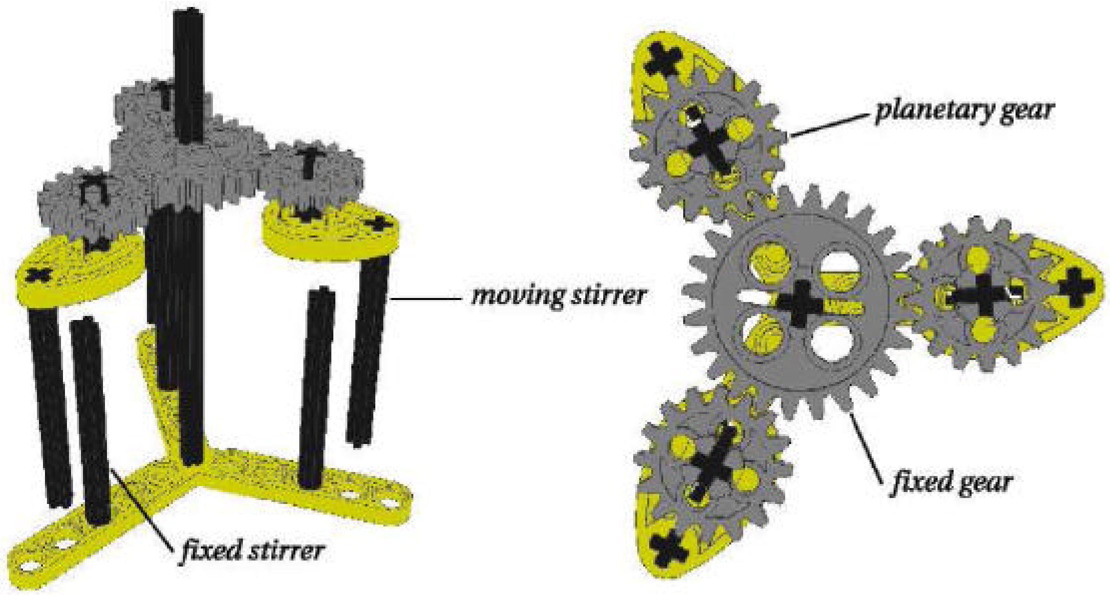
Kobayashi-Umeda suggested a simple mechanism realizing p.A. mixing.



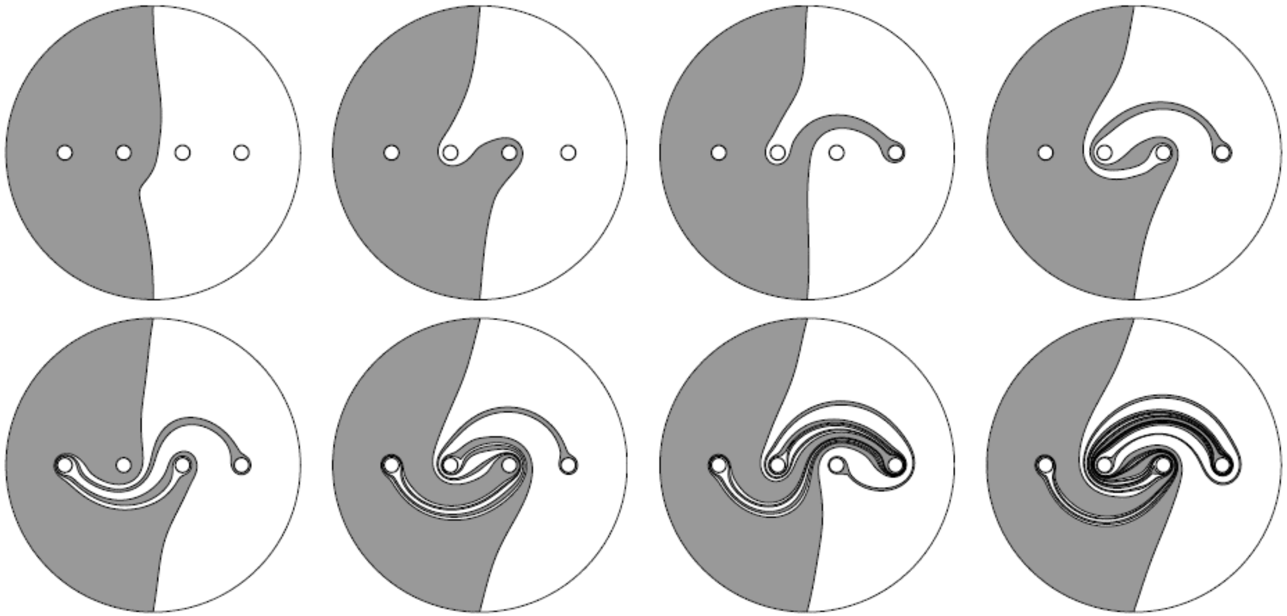
Kobayashi T. ,and Umeda, S, Realizing pseudo-Anosov egg beaters with simple mechanisms, Proc. of the Int. Workshop on Knot theory for Sci. Objects, March(2006), 97-109, 2007.

Thiffeault - Finn

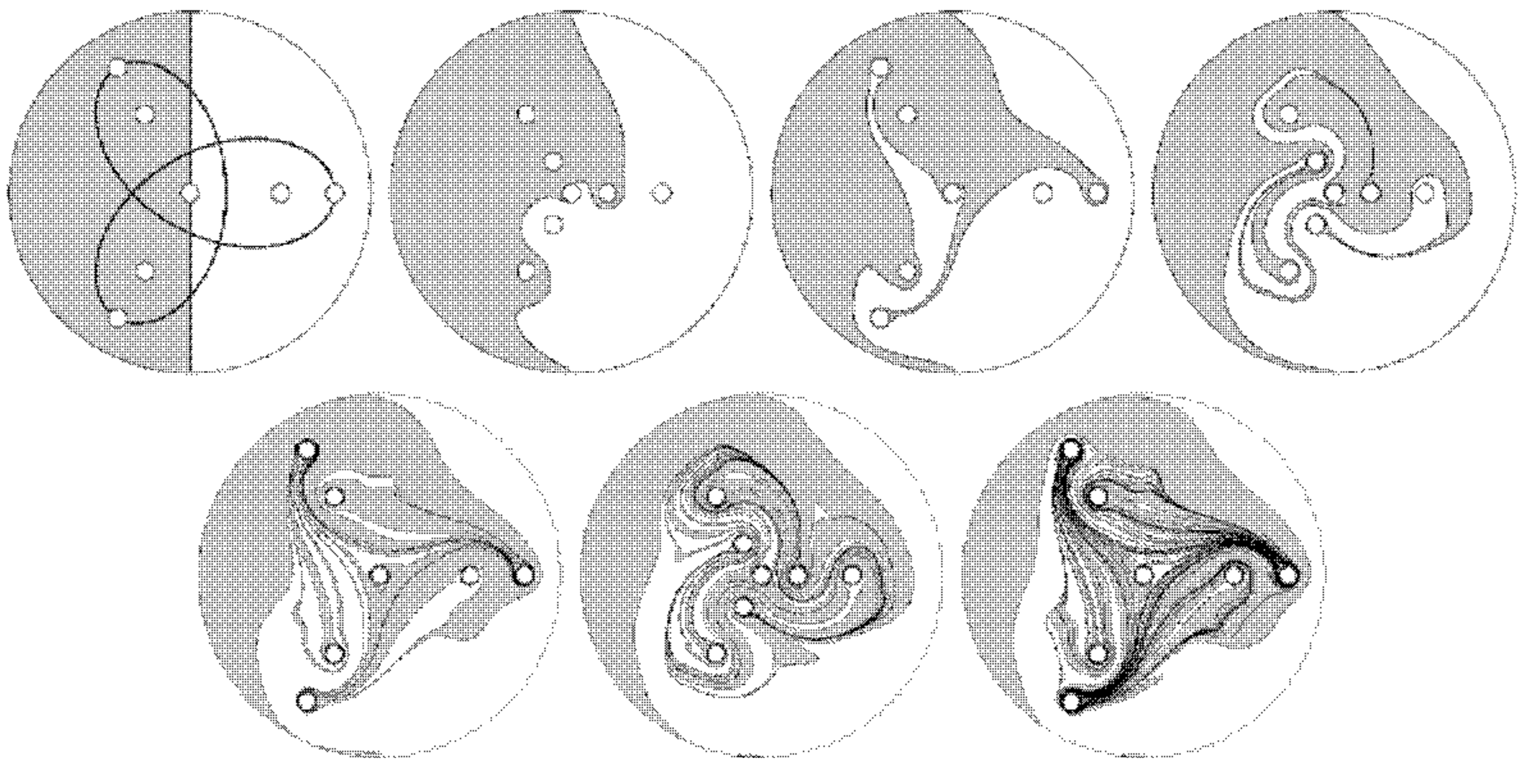
p.A. mixing with 6 (or 7) rods can stir much larger region than mixing with 3 rods.



Thiffeault,J.L., Finn, M.D., Topology, Braids, and Mixing in Fluids, Math, Phys. and Eng. Sci. 364 (2006), 3251 - 3266



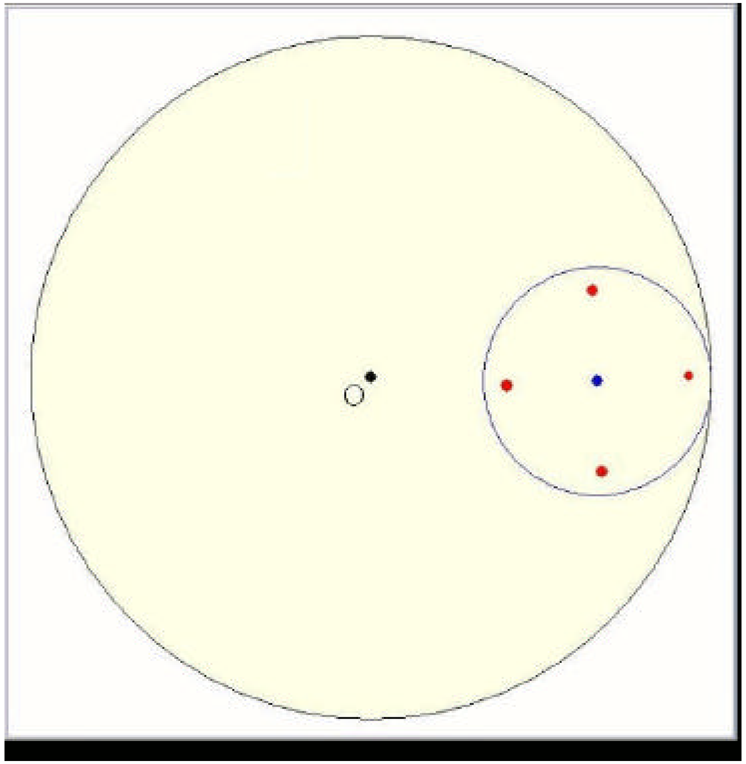
However, there is large region
which is not mixed at all.



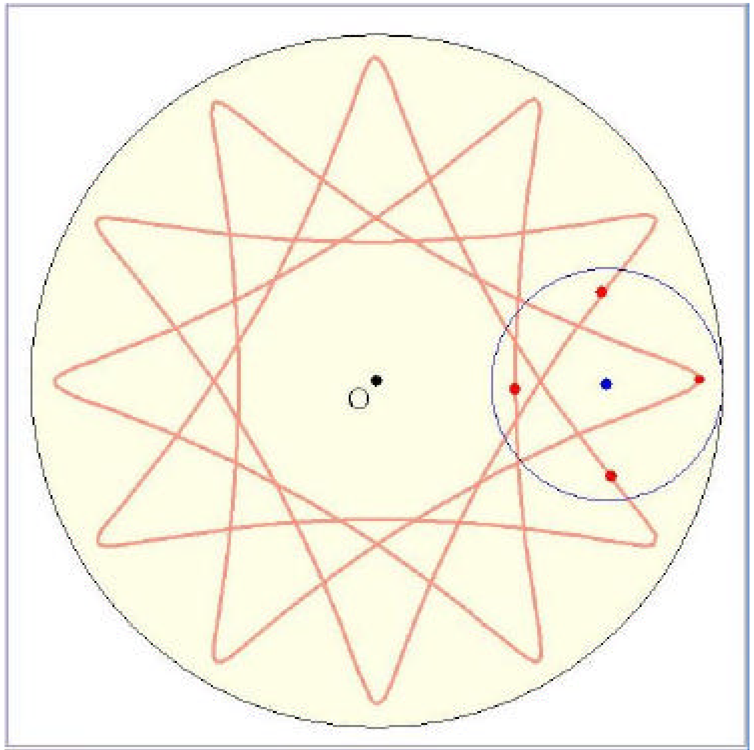
Thiffeault-Finn say :

“More rods will lead to a greater topological entropy, but will also complicate the apparatus.”

In this talk we introduce a mixing system using trochoid.



Outer circle : Inner circle = 3 : 1
Number of moving points = 4

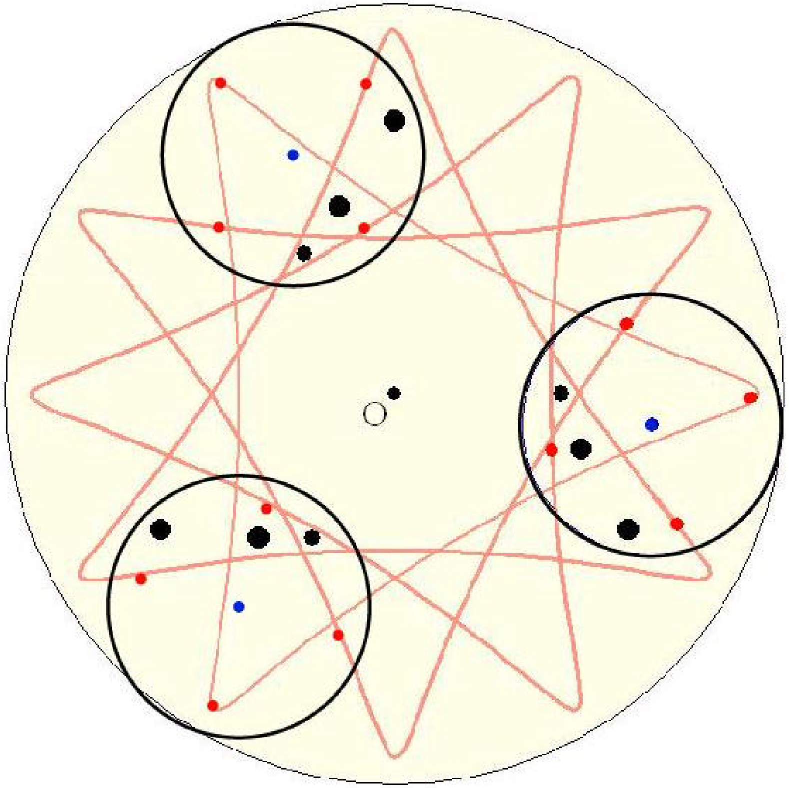


Place rods at the moving points.

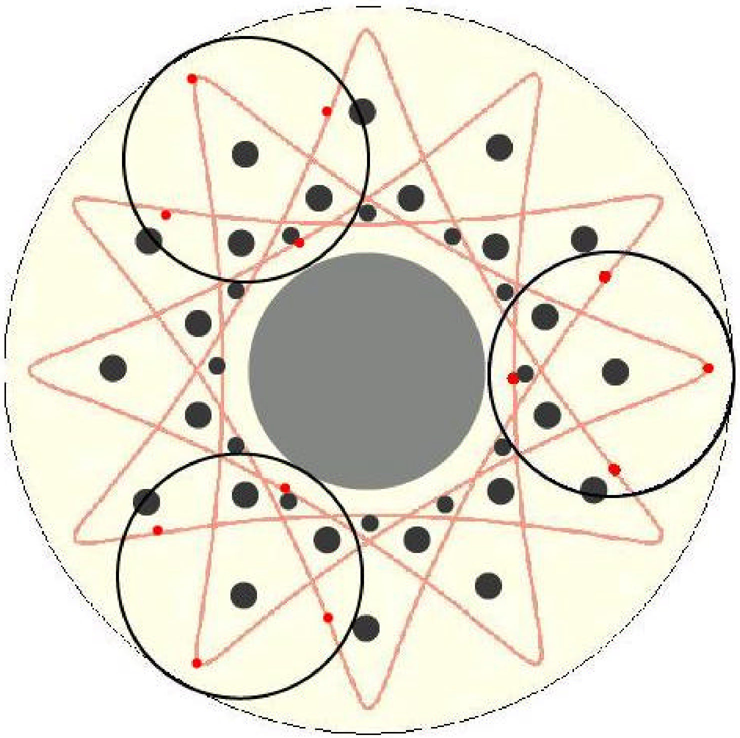
⇒the rods mix up fulids in the outer circle.

Furthermore:
Place obstacles as follows:

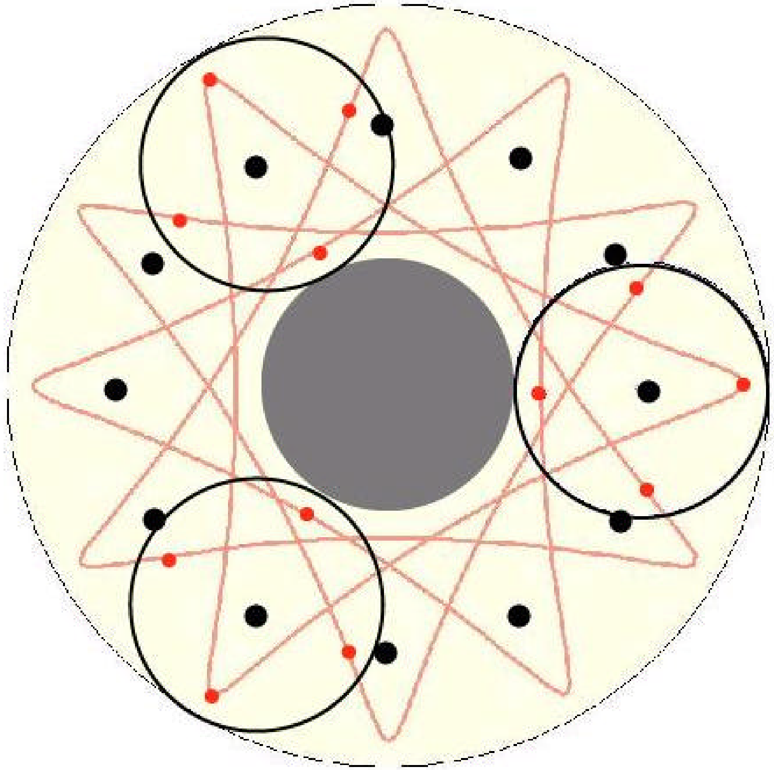
1.



2.



3.

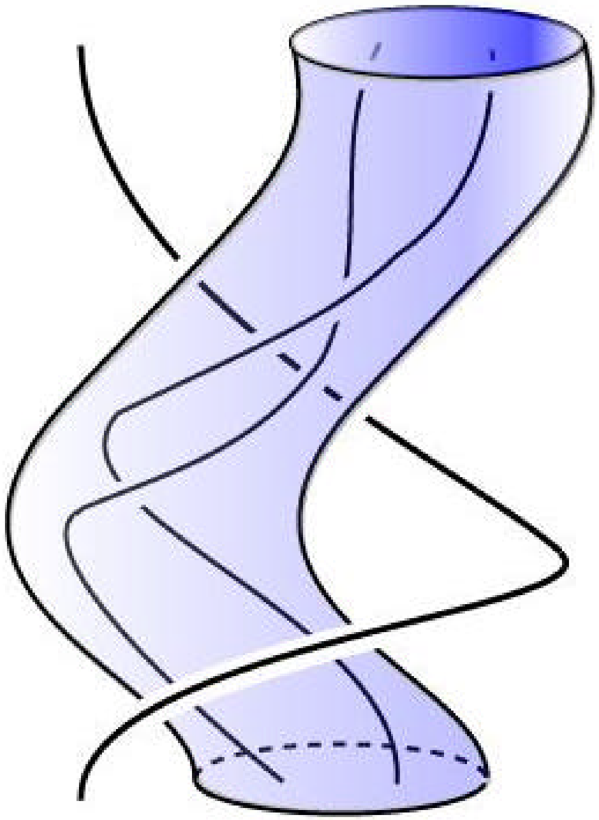


Fact:

These mixings are all of type p.A.

Proof :

Consider the links obtained by closing braids defined by the rods and obstacles.



Observation: If it is reducible,

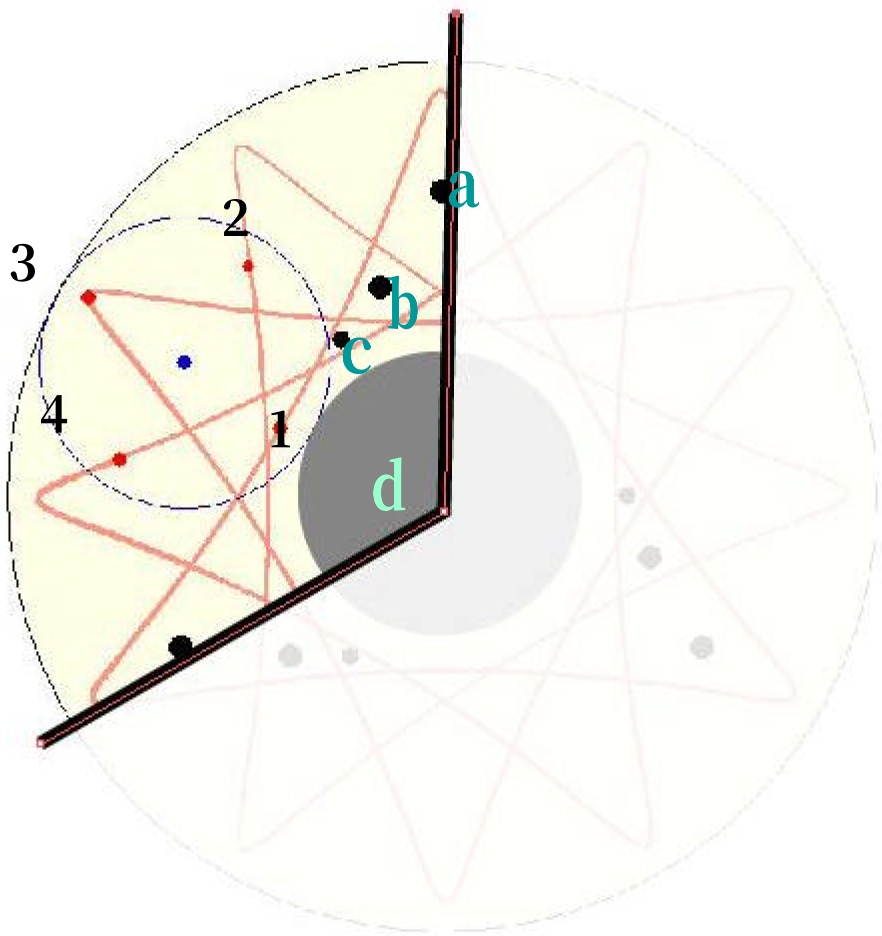
$$\mathrm{lk}(l_1, l_i) = \mathrm{lk}(l_2, l_i).$$

for any

l_1, l_2 : inside tube

l_i :outside tube

1. Fundamental region



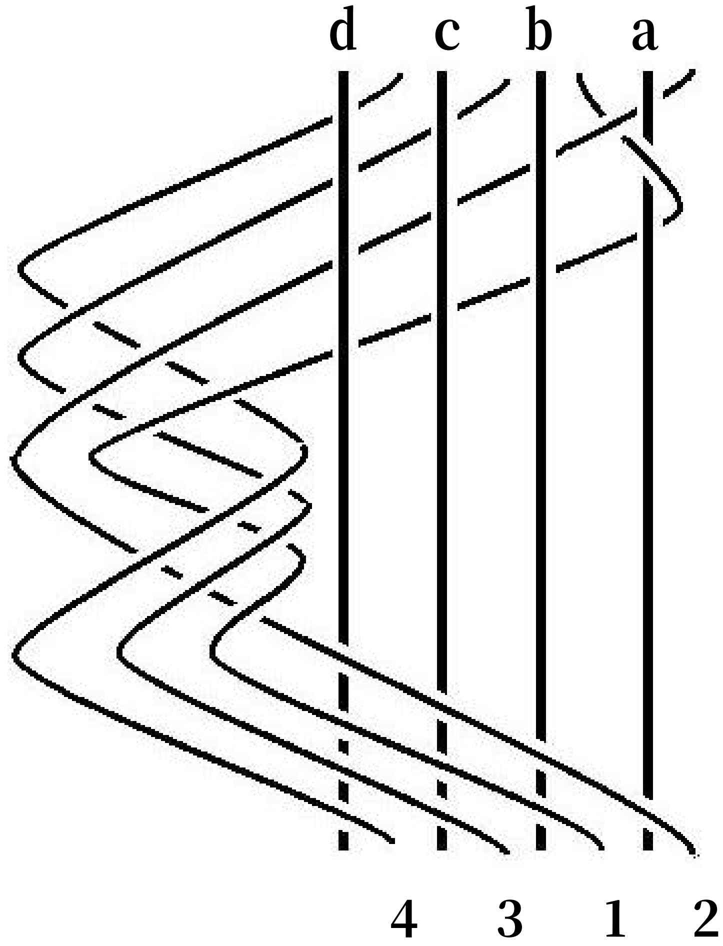


Table of Linking number

	a	b	c	d
1	1	1	1	1
2	0	1	1	1
3	0	0	1	1
4	0	0	0	1

lk(i, j) = -1 ($i, j \in \{1, 2, 3, 4\}$)

lk(x, y) = 0 ($x, y \in \{a, b, c, d\}$)

2. Fundamental region

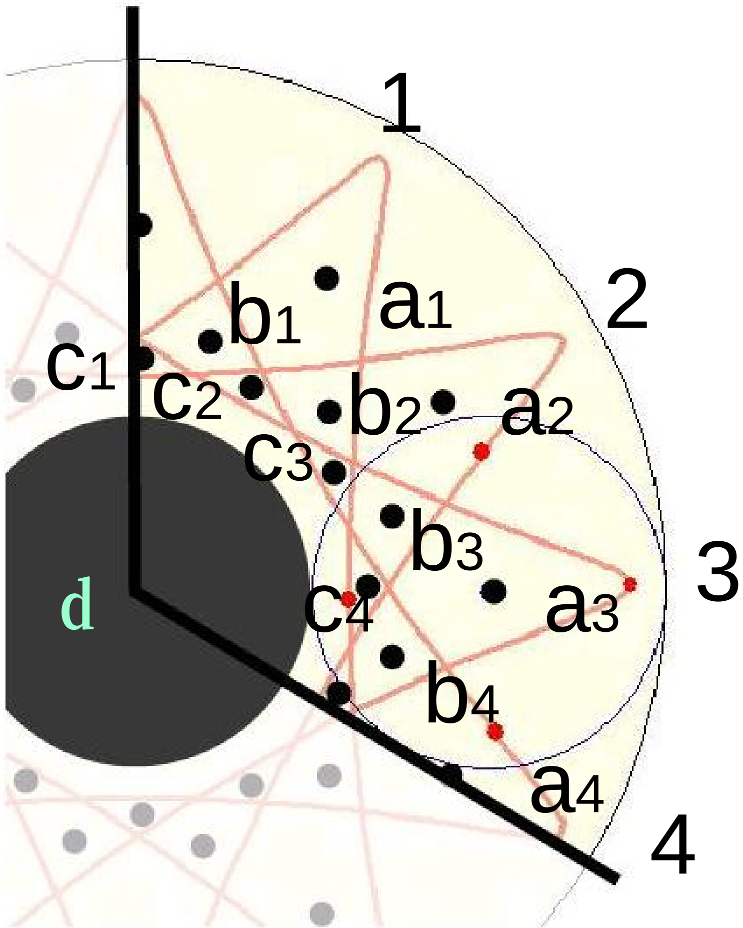


Table of linking numbers

	a_1	b_1	c_1	d
1	1	1	1	1

	a_2	b_2	c_2	d
1	0	1	1	1

	a_3	b_3	c_3	d
1	0	0	1	1

	a_4	b_4	c_4	d
1	0	0	0	1

etc.

3. Fundamental regions

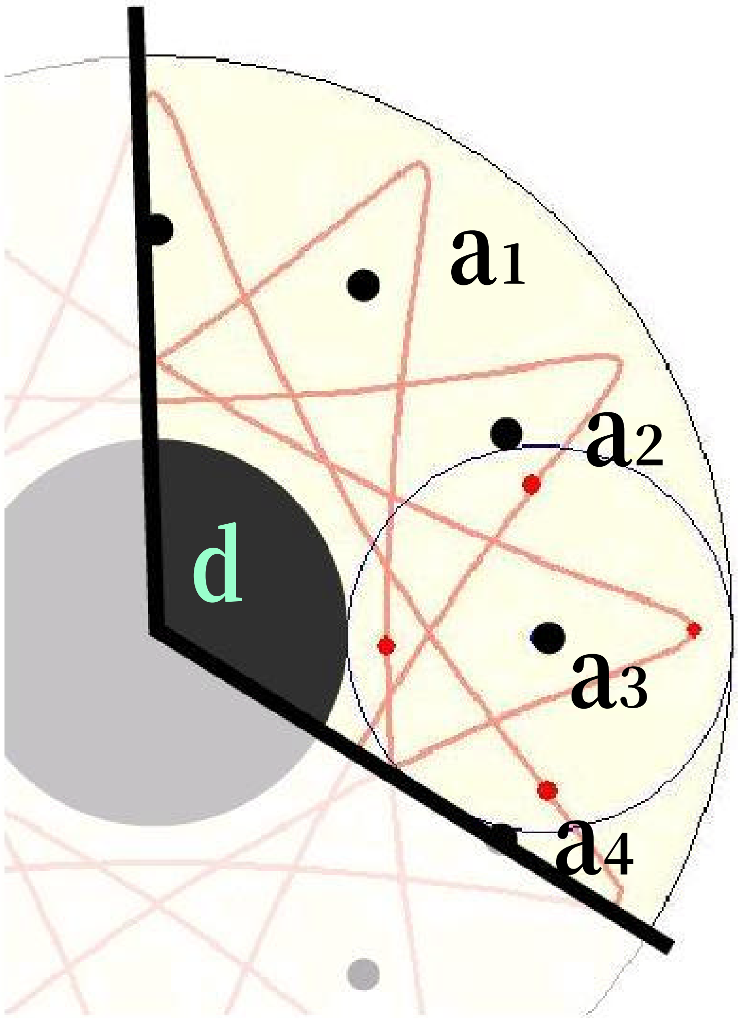


Table of linking numbers

	1	2	3	4
d	1	1	1	1
a_1	1	0	0	0
a_2	0	1	0	0
a_3	0	0	1	0
a_4	0	0	0	1